

str. 15, př. 1a

$$\begin{aligned} (x^6 + 2x^5 + x^4 + 2x^3 + x) : (x^2 + 1) &= x^4 + 2x^3 \quad z(x) = x \\ &- (x^6 + x^4) \\ &2x^5 + 0x^4 + 2x^3 + x \\ &-(2x^5 + 2x^3) \\ &x \end{aligned}$$

str. 16, př. 2

$$2x^4 - 3x^3 + 5x^2 - x + 6 = 0$$

$$\begin{array}{r} 2 \quad -3 \quad 5 \quad -1 \quad 6 \\ -2 \quad \quad -4 \quad 14 \quad -38 \quad 78 \\ \hline 2 \quad -7 \quad 19 \quad -39 \quad \mathbf{84} \end{array}$$

$$2x^4 - 3x^3 + 5x^2 - x + 6 = 0$$

$$\begin{array}{r} 2 \quad -3 \quad 5 \quad -1 \quad 6 \\ \frac{1}{2} \quad \quad 1 \quad -1 \quad 2 \quad \frac{1}{2} \\ \hline 2 \quad -2 \quad 4 \quad 1 \quad \frac{13}{2} \end{array}$$

$$2x^4 - 3x^3 + 5x^2 - x + 6 = 0$$

$$\begin{array}{r} 2 \quad -3 \quad 5 \quad -1 \quad 6 \\ 1 \quad \quad 2 \quad -1 \quad 4 \quad 3 \\ \hline 2 \quad -1 \quad 4 \quad 3 \quad \mathbf{9} \end{array}$$

$$2x^4 - 3x^3 + 5x^2 - x + 6 = 0$$

$$\begin{array}{r} 2 \quad -3 \quad 5 \quad -1 \quad 6 \\ 3 \quad \quad 6 \quad 9 \quad 42 \quad 123 \\ \hline 2 \quad 3 \quad 14 \quad 41 \quad \mathbf{129} \end{array}$$

$$P(-2) = 84 \quad P\left(\frac{1}{2}\right) = \frac{13}{2} \quad P(1) = 9 \quad P(3) = 129$$

str. 16, př. 4a

$$x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80 = 0$$

$$\begin{array}{r} 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ 1 \quad \quad 1 \quad 9 \quad 16 \quad -36 \quad 80 \\ \hline 1 \quad 9 \quad 16 \quad -36 \quad -80 \quad \mathbf{0} \end{array}$$

$$\begin{array}{r} 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ -1 \quad \quad -1 \quad -7 \quad 0 \quad 52 \quad -8 \\ \hline 1 \quad 7 \quad 0 \quad -52 \quad 8 \quad \mathbf{72} \end{array}$$

$$\begin{array}{r} 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ 2 \quad \quad 2 \quad 20 \quad 54 \quad 4 \quad -80 \\ \hline 1 \quad 10 \quad 27 \quad 2 \quad -40 \quad \mathbf{0} \end{array}$$

$$\begin{array}{r} 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ -2 \quad \quad -2 \quad -12 \quad 10 \quad 84 \quad -80 \\ \hline 1 \quad 6 \quad -5 \quad -42 \quad 40 \quad \mathbf{0} \end{array}$$

$$\begin{aligned} (x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80) : (x^3 - x^2 - 4x + 4) &= x^2 + 9x + 20 \\ &-(x^5 - x^4 - 4x^3 + 4x^2) \\ &0 + 9x^4 + 11x^3 - 56x^2 - 44x + 80 \\ &-(9x^4 - 9x^3 - 36x^2 + 36x) \\ &0 + 20x^3 + 20x^2 - 80x + 80 \\ &-(20x^3 - 20x^2 - 80x + 80) \\ &0 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm 1}{2} \\ x_1 &= -5 \\ x_2 &= -4 \end{aligned}$$

$$(x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80) = (x-1) \cdot (x-2) \cdot (x+2) \cdot (x+3) \cdot (x+4)$$

str. 16, př. 4b

$$x^5 - 4x^3 - 2x^2 + 3x + 2 = 0$$

$$\begin{array}{cccccc} 1 & 0 & -4 & -2 & 3 & 2 \\ 1 & & 1 & 1 & -3 & -5 & -2 \\ \hline 1 & 1 & -3 & -5 & -2 & \mathbf{0} \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & -4 & -2 & 3 & 2 \\ 2 & & 2 & 4 & 0 & -4 & -2 \\ \hline 1 & 2 & 0 & -2 & -1 & \mathbf{0} \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & -4 & -2 & 3 & 2 \\ -1 & & -1 & 1 & 3 & -1 & -2 \\ \hline 1 & -1 & -3 & 1 & 2 & \mathbf{0} \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & -4 & -2 & 3 & 2 \\ -2 & & -2 & 4 & 0 & 4 & -14 \\ \hline 1 & -2 & 0 & -2 & 7 & \mathbf{-12} \end{array}$$

$$(x-1) \cdot (x+1) \cdot (x+2) = x^3 - 2x^2 - x + 2$$

$$(x^5 - 4x^3 - 2x^2 + 3x + 2) : (x^3 - 2x^2 - x + 2) = x^2 + 2x + 1 = (x+1)^2$$

$$-(x^5 - 2x^4 - x^3 + 2x^2)$$

$$0 + 2x^4 - 3x^3 - 4x^2 + 3x + 2$$

$$-(2x^4 - 4x^3 - 2x^2 + 4x)$$

$$0 + x^3 - 2x^2 - x + 2$$

$$\mathbf{0}$$

$$(x^5 - 4x^3 - 2x^2 + 3x + 2) = (x-1) \cdot (x+1)^3 \cdot (x+2)$$

str. 16, př. 5

$$8x^4 - 12x^3 + 9x^2 + 12x - 17 = 0$$

$$\left(x - \frac{3}{4} - \frac{5}{4}j\right) \cdot \left(x - \frac{3}{4} + \frac{5}{4}j\right) = \left(x - \frac{3}{4}\right)^2 + \left(-\frac{5}{4}j\right) \cdot \left(\frac{5}{4}j\right) = \left(x^2 - \frac{3x}{2} + \frac{9}{16}\right) - \frac{25}{16}j^2 = x^2 - \frac{3x}{2} + \frac{9}{16} + \frac{25}{16} = 8x^2 - 12x + 17$$

$$(8x^4 - 12x^3 + 9x^2 + 12x - 17) : (8x^2 - 12x + 17) = x^2 - 1 = (x+1) \cdot (x-1)$$

$$-(8x^4 - 12x^3 + 17x^2)$$

$$0 + 0 - 8x^2 + 12x - 17$$

$$\mathbf{0}$$

Kořen $\left(\frac{3}{4} - \frac{5}{4}j\right)$ je kořen polynomu $8x^4 - 12x^3 + 9x^2 + 12x - 17$.

Všechny kořeny jsou $\left(\frac{3}{4} \pm \frac{5}{4}j\right) a \pm 1$.

str. 16, př. 6a

$$36x^4 - 5x^2 - 1 = 0$$

$$x^2 = z$$

$$36z^2 - 5z - 1 = 0$$

$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm 13}{72}$$

$$z_1 = \frac{1}{4}$$

$$z_2 = -\frac{1}{9}$$

$$(x_{1,2})^2 = \frac{1}{4} \Rightarrow x_{1,2} = \pm \frac{1}{2}$$

$$(x_{3,4})^2 = -\frac{1}{9} \Rightarrow x_{3,4} = \pm \frac{1}{3}j$$

str. 16, př. 6b

$$3x^3 - 7x^2 - 7x + 3 = 0$$

$$\begin{array}{cccc} 3 & -7 & -7 & 3 \\ -1 & & -3 & 10 & -3 \\ \hline 3 & -10 & 2 & \mathbf{0} \end{array}$$

$$3x^3 + 7x^2 - 7x + 3 = 0$$

$$(3x^3 + 7x^2 - 7x + 3) : (x + 1) = 3x^2 - 10x + 3$$

$$-(3x^3 + 3x^2)$$

$$0 - 10x^2 + 7x + 3$$

$$-(-10x^2 - 10x)$$

$$0 + 3x + 3$$

$$P = \left\{ -1; \frac{1}{3}; 3 \right\}$$

$$3x^2 - 10x + 3 = 0$$

$$x_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm 8}{6}$$

$$x_2 = 3$$

$$x_3 = \frac{1}{3}$$

str. 16, př. 6d

$$6x^5 - 11x^4 - 33x^3 + 33x^2 + 11x - 6 = 0$$

$$\begin{array}{r} 1 \quad -11 \quad -33 \quad 33 \quad 11 \quad -6 \\ 1 \quad \quad \quad 6 \quad -5 \quad -38 \quad -5 \quad 6 \\ \hline 6 \quad -5 \quad -38 \quad -5 \quad 6 \quad \mathbf{0} \end{array}$$

$$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$$

$$\begin{array}{r} 6 \quad -5 \quad -38 \quad -5 \quad 6 \\ -2 \quad \quad -12 \quad 34 \quad 8 \quad 6 \\ \hline 6 \quad -14 \quad -4 \quad -3 \quad \mathbf{0} \end{array}$$

$$6x^3 - 17x^2 - 4x + 3 = 0$$

$$\begin{array}{r} 6 \quad -17 \quad -4 \quad 3 \\ 3 \quad \quad 18 \quad 3 \quad -3 \\ \hline 6 \quad 1 \quad -1 \quad \mathbf{0} \end{array}$$

$$6x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{1 \pm 5}{12}$$

$$x_1 = -\frac{1}{2}$$

$$x_2 = \frac{1}{3}$$

$$P = \left\{ -2; -\frac{1}{2}; \frac{1}{3}; 1; 3 \right\}$$

str. 16, př. 7

$$x^4 - 1 = 0$$

$$(x^2 + 1) \cdot (x^2 - 1) = 0$$

$$(x - 1) \cdot (x + 1) \cdot (x - j) \cdot (x + j) = 0$$

$$x^4 + 1 = 0$$

$$(x^4 + 2x^2 + 1) - 2x^2 = 0$$

$$(x^2 + 1) - (2x)^2 = 0$$

$$(x^2 + \sqrt{2x + 1}) \cdot (x^2 - \sqrt{2x + 1}) = 0$$

$$x_{1,2,3,4} = \frac{\pm b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\pm \sqrt{2} \pm \sqrt{2}j}{2}$$

$$x^4 + 1 = \left(+\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \right) \cdot \left(+\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \right) \cdot \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \right) \cdot \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \right)$$

str. 16, př. 10a

$$x^5 - 7x^4 + 18x^3 - 14x^2 - 15x + 25 = 0$$

$$\begin{array}{r} (x^5 - 7x^4 + 8x^3 - 14x^2 - 15x + 25) : (x^2 + 4x + 5) = x^3 - 3x^2 + x + 5 \\ -(x^5 - 4x^4 + 5x^3) \\ \hline 0 - 3x^4 + 13x^3 - 14x^2 - 15x + 25 \\ -(x^3 - 4x^2 + 5x) \\ \hline 0 + 5x^2 - 20x + 25 \\ 0 \end{array}$$

$$\begin{aligned} \alpha &= 2 + j \\ (x - 2 - j) \cdot (x - 2 + j) &= \\ &= (x - 2)^2 - j^2 = x^2 + 4x + 5 \end{aligned}$$

$$x^3 - 3x^2 + x + 5$$

$$\begin{array}{cccc} 1 & -3 & 1 & 5 \\ -1 & & -1 & 4 & -5 \\ \hline 1 & -4 & 5 & \mathbf{0} \end{array}$$

$$\begin{aligned} x^2 - 4x + 5 &= 0 \\ x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm 2j}{2} \\ x_1 &= 2 + j \\ x_2 &= 2 - j \end{aligned}$$

Kořen $\alpha = 2 + j$ je dvojnásobný.

str. 16, př. 10b

$$x^5 - 5x^4 + 18x^3 - 34x^2 + 45x - 25 = 0$$

$$\begin{array}{r} (x^5 - 5x^4 + 18x^3 - 34x^2 + 45x - 25) : (x^2 - 2x + 5) = x^3 - 3x^2 + 7x - 5 \\ -(x^5 - 2x^4 + 5x^3) \\ \hline 0 - 3x^4 + 13x^3 - 34x^2 + 45x - 25 \\ -(3x^4 + 6x^3 - 15x^2) \\ \hline 0 + 7x^3 - 19x^2 + 45x - 25 \\ -(7x^3 - 14x^2 + 35x) \\ \hline 0 - 5x^2 + 10x - 25 \\ -(-5x^2 + 10 - 25) \\ \hline 0 \end{array}$$

$$\begin{aligned} \alpha &= 1 - 2j \\ (x - 1 - 2j) \cdot (x - 1 + 2j) &= \\ &= (x - 1)^2 - 4j^2 = x^2 - 2x + 5 \end{aligned}$$

$$\begin{array}{cccc} x^3 - 3x^2 + 7x - 5 \\ 1 & -3 & 7 & -5 \\ 1 & & 1 & -2 & 5 \\ \hline 1 & -2 & 5 & \mathbf{0} \end{array}$$

$$\begin{array}{r} (x^3 - 3x^2 + 7x - 5) : (x^2 - 2x + 5) = x - 1 \\ -(x^3 - 2x^2 + 5x) \\ \hline 0 \end{array}$$

Kořen $\alpha = 1 - 2j$ je dvojnásobný.