str. 72, př. 1
Určete exponenciální tvar komplexního čísla.
a) $z = -5 + 0i$

$$|z| = \sqrt{a^2 + b^2} = 5$$

$$\cos\varphi = \frac{x}{|z|} = -1 \Rightarrow \varphi = \pi$$

$$\sin\varphi = \frac{x}{|z|} = 0 \Rightarrow \varphi = \pi$$

$$z = -5 + 0i = 5e^{i\pi}$$

b)

$$z = 0 - 2i$$

$$|z| = \sqrt{a^2 + b^2} = 2$$

$$\cos\varphi = \frac{x}{|z|} = \frac{0}{2} \Rightarrow \varphi = \frac{\pi}{2}, \frac{3\pi}{2} = -\frac{\pi}{2}$$

$$\sin\varphi = \frac{x}{|z|} = \frac{-2}{2} \Rightarrow \varphi = \frac{3\pi}{2} = -\frac{\pi}{2}$$

$$z = 0 - 2i = 2e^{-i\frac{\pi}{2}}$$

c)

$$z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$|z| = \sqrt{a^2 + b^2} = 1$$

$$\cos\varphi = \frac{x}{|z|} = -\frac{1}{2} \Rightarrow \varphi = \frac{2}{3}\pi, \frac{4}{3}\pi$$

$$\sin\varphi = \frac{x}{|z|} = -\frac{\sqrt{3}}{2} \Rightarrow \varphi = \frac{4}{3}\pi, \frac{5}{3}\pi$$

$$z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = e^{i\frac{4}{3}\pi}$$

str. 73, př. 2
Určete algebraický tvar komplexního čísla
a)

$$\frac{5}{2}e^{i\frac{3}{2}\pi} = \frac{5}{2}\left(\cos\frac{3}{2}\pi + i.\sin\frac{3}{2}\pi\right)$$

$$\cos\varphi = \frac{a}{|z|} \Rightarrow a = \cos\varphi.|z| = 0.\frac{5}{2}$$

$$\sin\varphi = \frac{b}{|z|} \Rightarrow b = \sin\varphi.|z| = -1.\frac{5}{2} = -\frac{5}{2}$$

$$\frac{5}{2}e^{i\frac{3}{2}\pi} = \frac{5}{2}\left(\cos\frac{3}{2}\pi + i.\sin\frac{3}{2}\pi\right) = -\frac{5}{2}i$$

b)

$$e^{i\frac{\pi}{3}} = \cos\frac{\pi}{3} + i.\sin\frac{\pi}{3}$$

$$\cos\varphi = \frac{a}{|z|} \Rightarrow a = \cos\varphi.|z| = \frac{1}{2}.1 = \frac{1}{2}$$

$$\sin\varphi = \frac{b}{|z|} \Rightarrow b = \sin\varphi.|z| = \frac{\sqrt{3}}{2}$$

$$e^{i\frac{\pi}{3}} = \cos\frac{\pi}{3} + i.\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

c)

$$4e^{i\pi} = 4\left(\cos\pi + i.\sin\pi\right)$$

$$\cos\varphi = \frac{a}{|z|} \Rightarrow a = \cos\varphi.|z| = -4$$

$$\sin\varphi = \frac{b}{|z|} \Rightarrow b = \sin\varphi.|z| = 0.4 = 0$$

$$4e^{i\pi} = 4\left(\cos\pi + i.\sin\pi\right) = -4$$

str. 73, př. 3

$$i = i$$

$$i^2 = -1$$

$$i^3 = i^2.i = -i$$

$$i^4 = i^2.i^2 = 1$$

$$i^{35} = \left(i^4\right)^8.i^3 = 1.(-i) = -i$$

$$i^5 = i^4.i = i$$

$$i^2 = -1$$

$$i^{42} = \left(i^4\right)^{10}.i^2 = 1.(-1) = -1$$

$$i^{36} = \left(i^4\right)^9 = 1$$

str. 73, př. 4

a)

$$(1+i)(2+i)+(1+i)(1+2i) = 2+i+2i+i^2+1+2i+i+2i^2 = 2+3i-i^2+1+3i+2i^2 = 6i$$

b)

$$\frac{1+i}{1+2i} = \frac{1+i}{1+2i}.\frac{1-2i}{1-2i} = \frac{1-2i+i-2i^2}{1-4i^2} = \frac{3-i}{5}$$

str. 73, př. 5

$$z_1 = 2e^{i\frac{\pi}{6}}, z_2 = 3e^{i3\frac{\pi}{4}}$$

$$z_1.z_2 = 2e^{i\frac{\pi}{6}}.3e^{i3\frac{\pi}{4}} = 2.3e^{i\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)} = 6e^{i\frac{11}{12}\pi}$$

str. 73, př. 6

$$\mathrm{z}_1 = 4e^{i\pi}, z_2 = 2e^{i\frac{\pi}{6}}$$

$$\frac{z_1}{z_2} = \frac{4e^{i\pi}}{2e^{i\frac{\pi}{6}}} = \frac{4}{2}e^{i\left(\pi-\frac{\pi}{6}\right)} = 2e^{i\frac{5}{6}\pi}$$

str. 73, př. 7

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$\left(\sqrt{2}+i\sqrt{3}\right)^5 = 2^{\frac{5}{2}} + 5.2^{\frac{4}{2}}i\sqrt{3} + 10.2^{\frac{3}{2}}i^2 3 + 10.2.i^3.3\sqrt{3} + 5\sqrt{2}.i^4 9 + i^5.9\sqrt{3} =$$

$$= 4\sqrt{2} + 20i\sqrt{3} - 60\sqrt{2} - 60\sqrt{3}i + 45\sqrt{2} + i.9\sqrt{3} =$$

$$= -11\sqrt{2} - i31\sqrt{3}$$

str. 73, př. 8

$$(1+i)^4 = \left(\sqrt{2}e^{i\pi}\right)^4 = 4e^{i\pi} = 4(\cos\pi + i\sin\pi) = \cos\pi.|z| + \sin\pi.|z| = -1.4 + 0.4 = -4$$

str. 73, př. 9

a)

$$z = -1 - i = \sqrt{2}e^{i.\frac{5}{4}\pi+2\pi}$$

$$\varphi = \arccos\frac{-1}{\sqrt{2}} = \frac{3}{4}\pi, \frac{5}{4}\pi$$

$$\varphi = \arcsin\frac{-1}{\sqrt{2}} = -\frac{\pi}{4}, \frac{5}{4}\pi$$

$$\sqrt[3]{z} = \sqrt[6]{2}.e^{i.\frac{5}{3.4}\pi+\frac{2k\pi}{3}} = \sqrt[6]{2}.e^{i.\frac{5}{12}\pi+\frac{2k\pi}{3}}$$

b)

$$z = -1 = e^{i\pi}$$

$$\varphi = \arccos(-1) = \pi$$

$$\varphi = \arcsin(0) = 0, \pi$$

$$\sqrt[6]{z} = e^{i\frac{\pi+2k\pi}{6}} = e^{i\frac{\pi}{6}+\frac{k\pi}{3}}$$

$k=0,\quad e^{i\frac{\pi}{6}}=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$k=1,\quad e^{i\frac{\pi}{2}}=i$

$k=2,\quad e^{i\frac{5}{6}\pi}=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$k=3,\quad e^{i\frac{7}{6}\pi}=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$

$k=4,\quad e^{i\frac{3}{2}\pi}=-i$

$k=5,\quad e^{i\frac{11}{6}\pi}=\frac{\sqrt{3}}{2}-\frac{1}{2}i$

str. 73, př. 10

$z=\sqrt[n]{|z|}e^{i\frac{(2\pi+2k\pi)}{n}}$

a)

$z^4=16$

$z=2e^{i\frac{(2\pi+2k\pi)}{4}}=2e^{i\frac{\pi}{2}+k\frac{\pi}{2}}=2e^{ik\frac{\pi}{2}},k=0,1,2,3$

b)

$z^6=1-i$

$z^6=\sqrt{2}e^{i\frac{-\pi}{4}+2k\pi}$

$\varphi=\arccos\frac{1}{\sqrt{2}}=\frac{\pi}{4},-\frac{\pi}{4}$

$\varphi=\arcsin\frac{-1}{\sqrt{2}}=-\frac{\pi}{4},\frac{5}{4}\pi$

$z=\sqrt[12]{2}e^{-i\frac{\pi}{4.6}+\frac{2k\pi}{6}}=\sqrt[12]{2}e^{-i\frac{\pi}{24}+\frac{k\pi}{3}}$

$k=0,1,2,3,4,5$