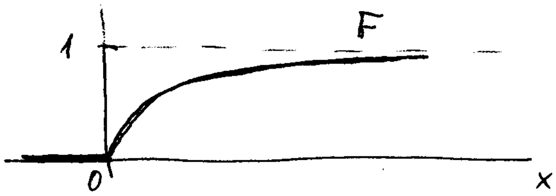
19.06.03

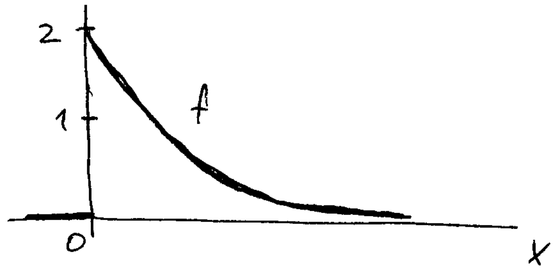
(1) $F(x) = \begin{cases} 1 - e^{-2x}, & x > 0 \\ 0, & \text{jinde} \end{cases}$



a) Hustota $f(x)$, grafy f, F

$f(x) = F'(x) = 2e^{-2x}, \quad x > 0$

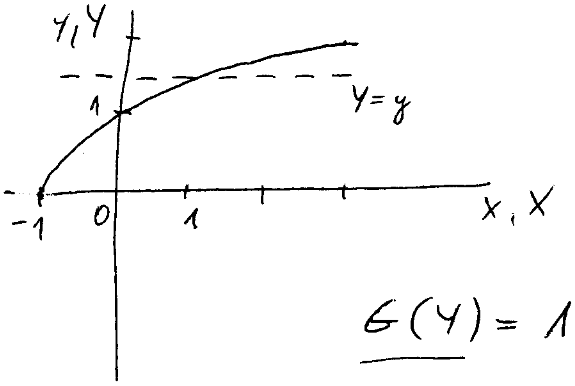
$= 0, \quad \text{jinde}$



b) $P(|X+1| \leq 2) = P(-3 \leq X \leq 1) = F(1) - F(-3) =$

$= F(1) - 0 = 1 - e^{-2} = \underline{\underline{1 - \frac{1}{e^2}}}$

c) Spočítat hustotu pro $Y = \sqrt{X+1}$



$x \in (0, \infty)$, $\underline{y \in (1, \infty)}$

$G(Y) = P(Y \leq y) = P(\sqrt{X+1} \leq y) =$

$= P(X \leq y^2 - 1) = F(y^2 - 1)$

$\underline{G(Y)} = 1 - e^{-2(y^2-1)} = \underline{1 - e^{-2y^2+2}}, \quad y > 1$

$\underline{g(y) = G'(y) = 4y\, e^{-2y^2+2}}, \quad \underline{y > 1}$

d) Kvantil $x_{0,5}$ a $E(X)$

$F(x_{0,5}) = 0,5 \Rightarrow 1 - e^{-2x_{0,5}} = 0,5 \Rightarrow e^{-2x_{0,5}} = 0,5 \Rightarrow$

$-2x_{0,5} = \ln 0,5 = \ln \frac{1}{2} \Rightarrow \underline{\underline{x_{0,5} = \frac{1}{2} \ln 2}}$

$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_0^{\infty} x \cdot 2e^{-2x}\, dx = \left| \begin{matrix} u = x & v' = e^{-2x} \\ u' = 1 & v = \frac{e^{-2x}}{-2} \end{matrix} \right| =$

$= \left[-\frac{1}{2} x e^{-2x} \right]_0^{\infty} + \frac{1}{2} \int_0^{\infty} e^{-2x}\, dx = \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^{\infty} =$

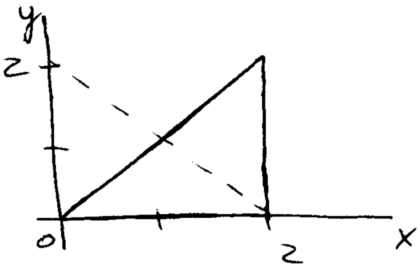
$= 0 - 0 + 0 + \frac{1}{4} = \underline{\underline{\frac{1}{4}}}$

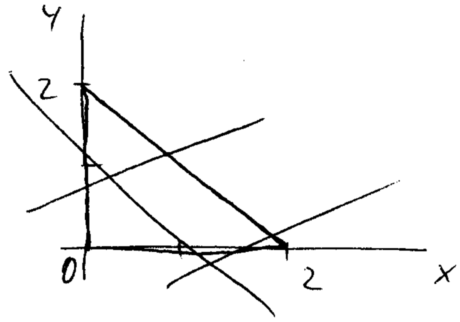
② Náhodný vektor X, Y ; sdružená hustota:

$$f(x,y) = \begin{cases} \frac{3}{16}(x^2+y) & , 0 < x < y < 2 \\ 0 & , \text{jinde} \end{cases}$$

~~(f se bude lišit, takto zadaná není hustota rozdělení)~~

a) $P(X+Y \leq 2) =$





$$= \int_0^1 \left(\int_y^{2-y} f(x,y)\,dx \right) dy =$$

$$= \int_0^1 \left(\int_y^{2-y} \frac{3}{16}(x^2+y)\,dx \right) dy = \frac{3}{16} \int_0^1 \left[\frac{x^3}{3} + xy \right]_y^{2-y} dy =$$

$$= \frac{3}{16} \int_0^1 \left(\frac{1}{3}(8 - 12y + 6y^2 - y^3) + 2y - y^2 - \frac{y^3}{3} - y^2 \right) dy =$$

$$= \frac{1}{16} \int_0^1 (8 - 12y + 6y^2 - y^3 + 6y - 3y^2 - y^3 - 3y^2)\,dy =$$

$$= \frac{1}{16} \int_0^1 (-2y^3 - 6y + 8)\,dy = \frac{1}{16} \left[-\frac{y^4}{2} - 3y^2 + 8y \right]_0^1 =$$

$$= \frac{1}{16}\left(-\frac{1}{2} - 3 + 8 \right) = \frac{1}{16} \cdot \frac{-1-6+16}{2} = \frac{1}{16} \cdot \frac{9}{2} = \underline{\underline{\frac{9}{32}}}$$

b) $f(y|1) \quad ? \quad = 0$ (kokotina)

$$f_1(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x \frac{3}{16}(x^2+y)\,dy = \frac{3}{16}\left[x^2 y + \frac{y^2}{2} \right]_0^x = \frac{3}{16}\left(x^3 - \frac{x^2}{2} \right)$$

$$f(y|x) = \frac{f(x,y)}{f_1(x)} = \frac{\frac{3}{16}(x^2+y)}{\frac{3}{16}(x^3 + \frac{x^2}{2})} = \underline{\frac{x^2+y}{x^3 + \frac{x^2}{2}}}$$

c) <u>Jsou závislé</u>

(Aby byly nezávislé, musela by f být kladná na obdélníku.)

③ Normální rozdělení $N(\mu, \sigma^2)$

a) Určete μ, σ ; $P(X \leq 50) = 0{,}8$, $P(X \leq 20) = 0{,}1$

$$\Phi\left(\frac{50-\mu}{\sigma}\right) = 0{,}8 \qquad \Phi\left(\frac{20-\mu}{\sigma}\right) = 0{,}1$$

$$\Phi\left(\frac{-20+\mu}{\sigma}\right) = 0{,}9$$

$$\left.\begin{aligned} 50-\mu &= \sigma \cdot u_{0,8} \\ -20+\mu &= \sigma \cdot u_{0,9} \end{aligned}\right\} \quad 30 = \sigma(u_{0,8} + u_{0,9})$$

$$\underline{\sigma} = \frac{30}{u_{0,8} + u_{0,9}} = \frac{30}{0{,}841 + 1{,}282} \doteq \underline{14{,}13}$$

$$\underline{\mu} = \sigma \cdot u_{0,9} + 20 = \underline{\underline{38{,}12}}$$

$$\Rightarrow N(38{,}12\,;\,1452{,}8)$$

b) $P(X \geq 30) = 1 - F(30) = 1 - \Phi\left(\frac{30 - 38{,}12}{14{,}13}\right) =$

$$= 1 - \Phi(-0{,}5746) = \cancel{1} - \cancel{1} + \Phi(0{,}5746) \doteq \underline{\underline{0{,}717}}$$

④ Komplexní funkce:

$$f(z) = 3\bar{z}(\mathrm{Re}(z) + 2\,\mathrm{Im}(z)) - 3jz^2$$

$$f(z) = 3(x - jy)(x + 2y) - 3j(x^2 + 2jxy - y^2) =$$
$$= 3(x^2 + 2xy - jxy - 2jy^2) - 3jx^2 + 6xy + 3jy^2 =$$
$$= \underline{3x^2} + \underline{6xy} - \underline{3jxy} - \underline{6jy^2} - \underline{3jx^2} + \underline{6xy} + \underline{3jy^2} =$$
$$= (3x^2 + 12xy) + j(-3x^2 - 3xy - 3y^2) = u + jv$$

$\underline{D_f = \mathbb{C}}$

$$\frac{\partial u}{\partial x} = 6x + 12y \qquad \frac{\partial v}{\partial y} = -3x - 6y$$

$$\frac{\partial u}{\partial y} = 12y \qquad \frac{\partial v}{\partial x} = -6x - 3y$$

$$\left.\begin{aligned} 6x + 12y &= -3x - 6y \\ 12y &= -6x - 3y \end{aligned}\right\}$$

$$\left.\begin{aligned} 9x &= -18y \\ 6x &= -15y \end{aligned}\right\}$$

$$\left.\begin{aligned} 3x &= -6y \\ 3x &= -\tfrac{15}{2}y \end{aligned}\right\}$$

$$\Leftarrow \quad x = y = 0$$

f(z) má derivaci pouze v bodě [0, 0]

$$f'(z) = \frac{\partial u}{\partial x} + j\frac{\partial v}{\partial x} =$$

$$\underline{f'(0 + 0j)} = 0 + j \cdot 0 = \underline{0}$$

$\underline{f(z)}$ není holomorfní.