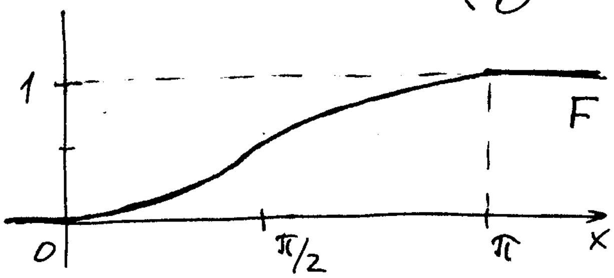
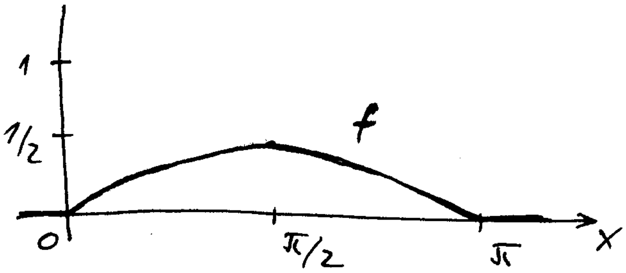
(13. 6. 03)

(1) $F(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{2}(1-\cos x) & , 0 \le x \le \pi \\ 1 & , x > \pi \end{cases}$

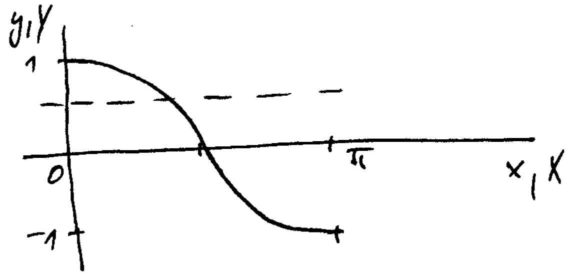
a) $f(x)$, grafy f, F

$$f(x) = F'(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{2}\sin x & , 0 \le x \le \pi \\ 0 & , x > \pi \end{cases}$$





b) $Y = \cos X$ $\qquad x \in (0, \pi)$, $y \in (-1, 1)$



$\underline{-1 \le y \le 1}$: $\underline{G(y)} = P(Y \le y) =$

$= P(\cos X \le y) = P(X \ge \arccos y) =$

$= 1 - F(\arccos y) = 1 - \frac{1}{2}(1 - \cos(\arccos y)) =$

$= 1 - \frac{1}{2}(1 - y) = \underline{\frac{1}{2}(1 + y)}$

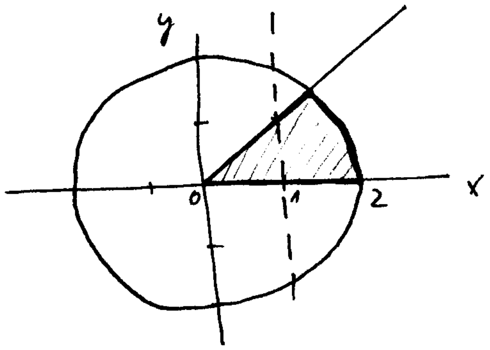
$\underline{g(y) =} \; G'(y) = \underline{\underline{\frac{1}{2}}}$, $\underline{-1 < y < 1}$

c) $\underline{E(X)} = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \frac{1}{2}\int_0^{\pi} (x - x\cos x)\,dx = \begin{vmatrix} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{vmatrix} =$

$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\pi} - \frac{1}{2}\left[x\sin x + \cos x\right]_0^{\pi} = \frac{\pi^2}{4} - \frac{1}{2}(0 - 1 - 0 - 1) =$

$= \underline{\underline{\frac{\pi^2}{4} + 1}}$

(2) $A = \{(x,y) ; x^2+y^2 \le 4 ; 0 \le y \le x\}$



Rovnoměrné rozdělení (zřejmě).

$$\iint_A a\,dx\,dy = a \cdot S_A = a \cdot \frac{\pi \cdot 2^2}{8} = a \cdot \frac{\pi}{2} = 1$$

$$\Rightarrow a = \frac{2}{\pi}$$

$$f(x,y) = \begin{cases} \frac{2}{\pi} &, (x,y) \in A \\ 0 &, \text{jinde} \end{cases}$$

a) $E(X) = \iint_{R^2} x \cdot f(x,y)\,dx\,dy = \iint_A \frac{2}{\pi} x\,dx = \left| \begin{matrix} \text{do pol.} \\ 0 \le \varphi \le \pi/4 \\ 0 \le \rho \le 2 \end{matrix} \right| =$

$$= \frac{2}{\pi} \int_0^{\pi/4} \left(\int_0^2 \rho^2 \cos\varphi\,d\rho \right) d\varphi = \frac{2}{\pi} \int_0^{\pi/4} \cos\varphi \left[\frac{\rho^3}{3} \right]_0^2 d\varphi =$$

$$= \frac{16}{3\pi} \left[\sin\varphi \right]_0^{\pi/4} = \frac{8\sqrt{2}}{3\pi}$$

b) $f_1(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_y^{\sqrt{4-x^2}} \frac{2}{\pi}\,dx = \frac{2}{\pi}\left(\sqrt{4-x^2} - y\right)$

c) $P(X \ge 1) = 1 - P(X \le 1) = 1 - \frac{2}{\pi} \cdot \frac{1}{2} =$

obsah $\triangle$ $[0,0][1,0][1,1]$

$$= 1 - \frac{1}{\pi}$$

(3) $N(\mu, 16)$, $P(X \geq 13) = 0{,}1$ $(\sigma = 4)$

$$P(X \geq 13) = 1 - P(X \leq 13) = 1 - F(13) =$$

$$= 1 - \Phi\left(\frac{13-\mu}{\sigma}\right) = 0{,}1$$

$$\Phi\left(\frac{13-\mu}{4}\right) = 0{,}9$$

$$\frac{13-\mu}{4} = u_{0,9} \quad \Rightarrow \underline{\mu} = 13 - 4u_{0,9} = 13 - 4 \cdot 1{,}28 \doteq \underline{\underline{7{,}88}}$$

$$\Rightarrow \underline{N(7{,}88 \,;\, 16)}$$

a) $\underline{P(|X-2| < 8)} = P(-6 \leq X \leq 10) = F(10) - F(-6) =$

$$= \Phi\left(\frac{10-7{,}88}{4}\right) - \Phi\left(\frac{-6-7{,}88}{4}\right) = \Phi(0{,}53) - 1 + \Phi(3{,}47) =$$

$$\doteq 0{,}70 - 1 + 1 \doteq \underline{\underline{0{,}7}}$$

b) $\underline{E(X^2)}$

$$D(X) = E(X^2) - (E(X))^2$$

$$\underline{E(X^2)} = D(X) + (E(X))^2 = \sigma^2 + \mu^2 \doteq \underline{\underline{78}}$$

(4) $\tanh z = 5 + 2j$

$$\tanh z = \frac{\sinh z}{\cosh z} = \frac{\frac{1}{2}(e^{z} - e^{-z})}{\frac{1}{2}(e^{z} + e^{-z})} = \frac{e^{2z} - 1}{e^{2z} + 1} = 5 + 2j$$

$$e^{2z} - 1 = (5+2j)e^{2z} + 5 + 2j$$

$$e^{2z}(4+2j) = -6 - 2j$$

$$e^{2z} = \frac{-6-2j}{4+2j} = \frac{1}{20}(-6-2j)(4-2j)$$

$$e^{2z} = \frac{1}{20}(-24 + 4j - 4) = \frac{1}{20}(4j - 28) = \frac{1}{5}(j - 7)$$

$$z = \frac{1}{2} \operatorname{Log}\left(\frac{1}{5}(-7+j)\right) = \frac{1}{2}\left[\ln \frac{\sqrt{50}}{5} + j\left(\pi - \operatorname{arctg}\frac{1}{7} + 2k\pi\right)\right] =$$

$$= \frac{1}{2} \ln \frac{\sqrt{50}}{5} + j\left(\frac{\pi}{2} - \frac{1}{2}\operatorname{arctg}\frac{1}{7} + k\pi\right), \quad k \in \mathbb{Z}$$

$$= \underline{\frac{1}{4} \ln 2 + j\left(\frac{\pi}{2} - \frac{1}{2}\operatorname{arctg}\frac{1}{7} + k\pi\right)}, \quad k \in \mathbb{Z}$$

⑤ a) Cauchyův vzorec pro integrál holomorf. funkce

Nechť f je holomorfní v oblasti G a $(\mathcal{C})$ je kladně orientovaná Jordanova cesta taková, že $(\mathcal{C} \cup \mathrm{Int}(\mathcal{C})) \subset G$. Potom pro každý bod $z_0 \in \mathrm{Int}\,\mathcal{C}$ je

$$\underbrace{\int_{(\mathcal{C})} \frac{f(z)}{z - z_0}\, dz = 2\pi j\, f(z_0)}$$

Význam – pro výpočet integrálů.

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b) Pravděpodob. funkce, pro jaké rozdělení, vlastnosti

– pro diskrétní rozdělení.

Funkce $p: R \to R$ def. vztahem $\underline{p(x) = P(X = x)}$, $\underline{x \in R}$

Vlastnosti: 1) $0 \leq p(x) \leq 1$, $x \in R$

2) $\sum_{x \in R} p(x) = 1$ (Hodnot nejvýše spočetné množství.)

3) $F(x) = P(X \leq x) = \sum_{x \leq t} P(X = t) = \sum_{x \leq t} p(t)$

4) $P(X = x) = p(x)$

– Po úsecích konstantní.

Binomické rozdělení

$$\underbrace{P(X = k) = p(k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

$k = 0, 1, 2, \ldots$

Poissonovo rozdělení

$$\underbrace{P(X = k) = p(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}}$$