str. 160 př. 1 (střední hodnota fce $\frac{1}{b-a}\int_a^b f(x)dx$)

$$\frac{1}{2}\int_{-1}^{1} 3xdx = \frac{1}{2}\left[\frac{3}{2}x^2\right]_{-1}^{1} = \frac{1}{2}\left(\frac{3}{2}-\frac{3}{2}\right) = 0$$

$$\frac{1}{1}\int_0^1 x^2 dx = 1\left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$$

$$\frac{1}{2}\int_0^2 e^x dx = \frac{1}{2}\int_0^2 e^x dx = \frac{1}{2}\left[e^x\right]_0^2 = \frac{1}{2}\left(e^2-1\right)$$

$$\frac{1}{\pi}\int_0^{\pi} \sin x = \frac{1}{\pi}\left[-\cos x\right]_0^{\pi} = \frac{1}{\pi}\left(1-(-1)\right) = \frac{2}{\pi}$$

str. 160 př. 2 (obsahy množin)

a)

$$\int_0^{\pi} \sin xdx - \int_0^{\pi} 0dx = \left[-\cos x\right]_0^{\pi} = 1-(-1) = 2$$

b)

$$\int_1^e \frac{1}{x}dx = \left[\ln x\right]_1^e = 1-0 = 1$$

c)

$$\int_a^b x^2 dx = \left[\frac{1}{3}x^3\right]_a^b = \frac{1}{3}b^3 - \frac{1}{3}a^3 = \frac{1}{3}\left(b^3-a^3\right)$$

d)

$$\int_{-\infty}^0 e^x = \left[e^x\right]_{-\infty}^0 = 1$$

$$\lim_{x\to 0} e^x - \lim_{x\to -\infty} e^x = 1-0 = 1$$

$$\int_0^{\infty} e^{-x} = \left[-e^x\right]_0^{\infty}$$

$$\lim_{x\to\infty}\left(-e^{-x}\right) - \lim_{x\to 0}\left(-e^{-x}\right) = 0-(-1) = 1$$

$$\int_{-\infty}^{\infty} e^{-|x|}dx = \int_{-\infty}^0 e^{-x} + \int_0^{\infty} e^{-x} = 1+1 = 2$$

e)

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}dx = \left[\arctan x\right]_{-\infty}^{\infty}$$

$$\lim_{x\to\infty}\left(\arctan x\right) - \lim_{x\to -\infty} \arctan x = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

str. 160 př. 3 (obsahy plochy)

a)

$$x^2 - 2x = 0$$

$$x.(x-2) = 0$$

$$x \in \langle 0,2 \rangle$$

$$\int_0^2 x^2 - 2x dx = \left[\frac{1}{3}x^3 - x^2 \right]_0^2 = \left| \frac{8}{3} - \frac{12}{3} \right| = \frac{4}{3}$$

b)

$$x^4 = x$$

$$x^4 - x = 0$$

$$x^3 (x-1) = 0$$

$$x \in \langle 0,1 \rangle$$

$$\int_0^1 x dx - \int_0^1 x^4 dx = \left[\frac{1}{2}x^2 \right]_0^1 - \left[\frac{1}{5}x^5 \right]_0^1 = \frac{1}{2} - \frac{1}{5} = \frac{5-2}{10} = \frac{3}{10}$$

c)

$$x^2 = \sqrt{x}$$

$$x^4 - x = 0$$

$$x^3 (x-1) = 0$$

$$x \in \langle 0,1 \rangle$$

$$\int_0^1 \sqrt{x} dx - \int_0^1 x^2 dx = \left[\frac{2}{3}x^{\frac{3}{2}} \right]_0^1 - \left[\frac{1}{3}x^3 \right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$