str. 142 př. 1

a)

$$\int_0^2 \left(3x^2-2x\right)dx=\left[x^3-x\right]_0^2=8-4=4$$

$$\int_1^2 \frac{dx}{x^2}=\left[-\frac{1}{x}\right]_1^2=-\frac{1}{2}-\left(-\frac{1}{1}\right)=\frac{1}{2}$$

$$\int_1^e \frac{dx}{x}=\left[\ln x\right]_1^e=\ln e-\ln 1=1-0=1$$

$$\int_2^6 \frac{dx}{x}=\left[\ln x\right]_2^6=\ln 6-\ln 2=\ln\frac{6}{2}=\ln 3$$

$$\int_1^4 \sqrt{x}dx=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4=\frac{16}{3}-\frac{2}{3}=\frac{14}{3}$$

$$\int_{-7}^0 \frac{2}{\sqrt[3]{x-1}}dx=-\left[3\sqrt[3]{\left(x-1\right)^2}\right]_{-7}^0=-\left(3\sqrt[3]{\left(-7-1\right)^2}\right)+\left(3\sqrt[3]{\left(0-1\right)^2}\right)=-12+3=-9$$

$$\int_0^\pi \sin 6x dx=\left[\frac{1}{6}\cos 6x\right]_0^\pi=0$$

$$\int_{-\pi/2}^{\pi/2} \cos\left(\frac{x}{2}\right)dx=2\left[2\sin\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}=4\sin\frac{\pi}{4}=2\sqrt{2}$$

$$\int_0^1 3^x\,dx=\left[\frac{3^x}{\ln 3}\right]_0^1=\frac{3}{\ln 3}-\frac{1}{\ln 3}=\frac{2}{\ln 3}$$

str. 142 př. 2

a)

$$\int_0^2 \left|x-1\right|dx=\int_0^1\left(1-x\right)dx+\int_1^2\left(x-1\right)dx=\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{x}-x\right]_1^2=\left(1-\frac{1}{2}\right)+\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)=1$$

b)

$$\int_{-2}^3 \left|x^2-1\right|dx=\int_{-2}^{-1}\left(x^2-1\right)dx+\int_{-1}^1\left(1-x^2\right)dx+\int_1^3\left(x^2-1\right)dx=\left[\frac{x^3}{3}-x\right]_{-2}^{-1}+\left[x-\frac{x^3}{3}\right]_{-1}^1+\left[\frac{x^3}{3}-x\right]_1^3=$$

$$=\left(-\frac{1}{3}+\frac{3}{3}\right)-\left(-\frac{8}{3}+\frac{6}{3}\right)+\left(\frac{3}{3}-\frac{1}{3}\right)-\left(-\frac{3}{3}+\frac{1}{3}\right)+\left(\frac{27}{3}-\frac{9}{3}\right)-\left(\frac{1}{3}-\frac{3}{3}\right)=\frac{28}{3}$$

str. 142 př. 3 (per-partes)

a)

$$\int_0^\pi \left(2x+1\right)\sin\left(\frac{x}{2}\right)=\left[8\sin\left(\frac{x}{2}\right)-4x\cos\left(\frac{x}{2}\right)-2\cos\left(\frac{x}{2}\right)\right]_0^\pi=\left(8-4\pi.0-2.0\right)-\left(8.0-4.0.1-2\right)=10$$

b)

$$\int_0^\pi \left(4x-1\right)\cos\left(2x\right)dx=\left[2x\sin\left(2x\right)-\frac{1}{2}\sin\left(2x\right)+\cos\left(2x\right)\right]_0^\pi=\left(2\pi.0-\frac{1}{2}.0+1\right)-\left(2.0.0-\frac{1}{2}.0+1\right)=0$$

c)

$$\int_{-1}^0 \left(3x+2\right)e^{3x}dx=\left[\frac{1}{3}\left(3x+1\right)e^{3x}\right]_{-1}^0=\left(\frac{1}{3}\right)-\left(-\frac{2}{3}e^{-3}\right)=\frac{1}{3}\left(1+2e^{-3}\right)$$

d)

$$\int_0^{\pi} x^2 \cos(x)\,dx = \left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\pi} = \left(\pi^2.0 + 2\pi(-1) - 2.0\right) - \left(0.0 + 2.0.1 - 2.0\right) = -2\pi$$

e)

$$\int_{-1}^{0} x^3 e^{-x} dx = \left[\left(-e^{-x}\right)\left(x^3 + 3x^2 + 6x + 6\right)\right]_{-1}^{0} = -\left(0+0+0+6\right) - \left(-e\left(-1+3-6+6\right)\right) = 2e - 6$$

f)

$$\int_0^1 x\arctan(x)\,dx = \left[\frac{1}{2}\arctan(x)\,x^2 - \frac{1}{2}x + \frac{1}{2}\arctan(x)\right]_0^1 = \left(\frac{1}{2}\frac{\pi}{4}.1 - \frac{1}{2} + \frac{1}{2}\frac{\pi}{4}\right) - \left(0-0+0\right) = \frac{\pi}{4} - \frac{1}{2}$$

g)

$$\int_0^1 \arcsin(x)\,dx = \left[x.\arcsin(x) + \sqrt{1-x^2}\right]_0^1 = \left(1.\frac{\pi}{2} + 0\right) + \left(0 + \sqrt{1}\right) = \frac{\pi}{2} + 1$$

str. 142 př. 4 (rozklad na parc. zlomky)

a)

$$\int_{-2}^{-1} \frac{x+1}{x^2(x-1)}\,dx = \int_{-2}^{-1} -\frac{1}{x^2} - \frac{2}{x} + \frac{2}{(x+1)}\,dx = \left[\frac{1}{x} - 2\ln|x| + 2\ln|x+1|\right]_{-2}^{-1} = \left(-1 - 2.0 + 2\ln 2\right) - \left(-\frac{1}{2} - 2\ln 2 + 2\ln 3\right) =$$

$$= 2\ln\frac{4}{3} - \frac{1}{2}$$

b)

$$\int_0^1 \frac{x^2+3x}{(x+1)(x^2+1)}dx = -\int_0^1 \frac{1}{x+1}\,dx + \int_0^1 \frac{2x}{x^2+1}\,dx + \int_0^1 \frac{1}{x^2+1}dx = \left[-\ln|x+1| + \ln\left|x^2+1\right| + \arctan(x)\right]_0^1 =$$

$$= \left(-\ln 2 + \ln 2 + \frac{\pi}{4}\right) - \left(-0+0+0\right) = \frac{\pi}{4}$$

c)

$$\int_{-2}^{3} \frac{2x^3 - 3x^2 - 20x - 14}{x^2 - x - 12}dx = \int_{-2}^{3}\left(2x - 1 - \frac{2}{x-4} + \frac{5}{x+3}\right)dx = \left[x^2 - x - 2\ln|x-4| + 5\ln|x+3|\right]_{-2}^{3} =$$

$$= \left(9 - 3 - 0 + 5\ln 6\right) - \left(4 + 2 - 2\ln 6 + 0\right) = 7\ln 6$$

d)

$$\int_3^4 \frac{x^2 - 2x - 4}{x^3 - 4x^2 + 4x}dx = \int_3^4 -\frac{1}{x} - \frac{2}{(x-2)^2} + \frac{2}{x-2}\,dx = \left[-\ln|x| + \frac{2}{x-2} + 2\ln|x-2|\right]_3^4 =$$

$$= \left(-\ln 4 + 1 + 2\ln 2\right) - \left(-\ln 3 + 2 + 0\right) = \ln 3 - 1$$

e)

$$\int_3^5 \frac{x-2}{x^2 - 6x + 13}\,dx = \frac{1}{2}\int_3^5 \frac{2x-6}{x^2 - 6x + 13}\,dx + \int_3^5 \frac{1}{x^2 - 6x + 13}\,dx$$

$$\frac{1}{2}\int_3^5 \frac{2x-6}{x^2 - 6x + 13}dx = \frac{1}{2}\left[\ln\left|x^2 - 6x + 13\right|\right]_3^5$$

$$\int_3^5 \frac{1}{x^2 - 6x + 13}\,dx = \frac{1}{4}\int_3^5 \frac{1}{\left(\frac{x-3}{2}\right)^2 + 1} = \frac{1}{4}\left[\arctan\left(\frac{x-3}{2}\right)\right]_3^5$$

$$\int_3^5 \frac{x-2}{x^2 - 6x + 13}\,dx = \frac{1}{2}\left[\ln\left|x^2 - 6x + 13\right| + \frac{1}{4}\arctan\left(\frac{x-3}{2}\right)\right]_3^5 = \left(\frac{1}{2}\ln 8 + \frac{\pi}{8}\right) - \left(\ln 4 + \frac{1}{2}.0\right) = \frac{1}{2}\ln 2 + \frac{\pi}{8}$$

f)

$$\int_0^1 \frac{dx}{4x^2+4x+5} = \frac{1}{4}\int_0^1 \frac{dx}{\left(\frac{x+1}{2}\right)^2+1} = \left[\frac{1}{4}\arctan\left(\frac{x+1}{2}\right) - \frac{1}{4}\arctan\left(\frac{x+1}{2}\right)\right] = \frac{1}{4}\arctan\left(\frac{3}{2}\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\right)$$

str. 142 př. 5 (substituce)

a)

$$\int_1^3 \frac{x}{\sqrt[3]{x^2-1}}dx = \left| \begin{matrix} t^2 = x^3+1 \\ x = \sqrt{t^3+1} \\ dx = \frac{3t^2}{2\sqrt{t^3+1}}dt \end{matrix} \right| = \int_1^3 \frac{\sqrt{t^3+1}}{t}.\frac{3t^2}{2\sqrt{t^3+1}}dt = \frac{3}{2}\int_1^3 tdt = \frac{3}{4}t^2 = \left[\frac{3}{4}\sqrt[3]{\left(x^2-1\right)^2}\right]_1^3 = 3-0=3$$

b)

$$\int_0^2 \frac{x^2}{\sqrt{x^3+1}}dx = \left| \begin{matrix} t^2 = x^3+1 \\ x = \sqrt[3]{t^2-1} \\ dx = \frac{2t}{3\left(t^2-1\right)^{\frac{2}{3}}} \end{matrix} \right| = \int_0^2 \frac{\left(t^2-1\right)^{\frac{2}{3}}}{t}.\frac{2t}{3\left(t^2-1\right)^{\frac{2}{3}}}dt = \frac{2}{3}\int_0^2 1dt = \frac{2}{3}t = \left[\frac{2}{3}\sqrt{x^3+1}\right]_0^2 = 2-\frac{2}{3} = \frac{4}{3}$$

c)

$$\int_0^\pi tg\left(\frac{x}{3}\right)dx = \int_0^\pi \frac{\sin(x/3)}{\cos(x/3)} = \left| \begin{matrix} t = \cos(x/3) \\ \sin x/3 = -3dt \end{matrix} \right| = -3\int_0^\pi \frac{1}{t}dt = -3\ln t = -3\ln\left(\cos\left(\frac{x}{3}\right)\right) = -3\ln\left(\frac{1}{2}\right) = 3\ln 2$$

str. 142 př. 6 (substituce)

a)

$$\int_0^{\ln 2} \frac{e^x-1}{e^x+1} = \left| \begin{matrix} e^x = t \\ x = \ln t \\ dx = \frac{1}{x}dt \end{matrix} \right| = \int_0^{\ln 2} \frac{t-1}{t+1}.\frac{1}{t}dt = \int_0^{\ln 2} -\frac{1}{t} + \frac{2}{t+1}dt = \left[-\ln e^x + 2\ln e^x + 1\right]_0^{\ln 2} =$$

$$= \left(-\ln 2 + 2\ln 3\right) - \left(0 + 2\ln 2\right) = \ln 9 - \ln 4 - \ln 2 = \ln\frac{9}{8}$$

b)

$$\int_0^e \frac{dx}{x\left(\ln^2 x+1\right)} = \left| \begin{matrix} t = \ln x \\ x = e^t \\ dx = e^t dt \end{matrix} \right| = \int_0^e \frac{1}{e^t\left(t^2+1\right)}e^t dt = \int_0^e \frac{1}{t^2+1}dt = \left[\arctan\left(\ln x\right)\right]_0^e = \frac{\pi}{4}$$

c)

$$\int_0^{\pi/2} \sin^2(x).\cos(x)dx = \left| \begin{matrix} t = \sin x \\ \cos x dx = dt \end{matrix} \right| = \int_0^{\pi/2} t^2 dt = \frac{1}{3}t^3 = \left[\frac{1}{3}\sin^3 x\right]_0^{\pi/2} = \frac{1}{3}$$

d)

$$\int_{\pi/2}^{2\pi} \frac{\sin x}{\cos^2 x - 2\cos x + 2} dx = \begin{vmatrix} t = \cos x \\ \sin x dx = -dt \end{vmatrix} = -\int_{\pi/2}^{2\pi} \frac{1}{t^2 - 2t + 2} dt = -\int_{\pi/2}^{2\pi} \frac{1}{(t-1)^2 + 1} dt =$$

$$= \left[-\arctan(\cos x)\right]_{\pi/2}^{2\pi} = -(0) + \left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

e)

$$\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}} dx = \begin{vmatrix} t^2 = x \\ dx = 2tdt \end{vmatrix} = \int_0^1 \frac{t}{1+t} 2tdt = \int_0^1 \frac{2t^2}{1+t} dt = \int_0^1 2t - 2 + \frac{2}{t+1} dt = t^2 - 2t + 2\ln|t+1| =$$

$$\left[x - 2\sqrt{x} + 2\ln\left|\sqrt{x}+1\right|\right]_0^1 = (1 - 2 + 2\ln 2) - (0) = 2\ln 2 - 1$$