str. 125 př. 1 (pomocí rozkladu na parciální zlomky)

a)

$$\int \frac{x^3-2x+5}{x^2-x-2}dx = \int x+1+\frac{x+7}{(x-2).(x+1)}dx = \int x+1+\frac{3}{(x-2)}-\frac{2}{(x-1)}dx = \frac{x^2}{2}+x+3\ln|x-2|-2\ln|x-1|+C =$$

$$\frac{x^2}{2}+x+\ln\frac{|x-2|^3}{(x-1)^2}+C, x\neq -1, x\neq 2$$

b)

$$\int \frac{2x+1}{(x-1)(x^2+3x+2)}dx = \int \frac{2x+1}{(x-1)(x+1)(x+2)}dx = \int \frac{1}{2(x-1)}+\frac{1}{2(x+1)}-\frac{1}{(x+2)}dx =$$

$$= \frac{1}{2}\ln|x-1|+\frac{1}{2}\ln|x+1|-\ln|x+2|+C = \frac{1}{2}\ln|x^2-1|-\ln|x+2|+C, x\neq\{-2,-1,1\}$$

c)

$$\int \frac{x^2+7x+1}{(x-1)(x^2+x-2)}dx = \int \frac{x^2+7x+1}{(x-1)^2(x+2)}dx = \int \frac{3}{(x-1)^2}+\frac{2}{(x-1)}-\frac{1}{(x+2)}dx =$$

$$-\frac{3}{x-1}+2\ln|x-1|-\ln|x+2|+C = -\frac{3}{x-1}+\ln\frac{(x-1)^2}{|x+2|}+C, x\neq\{-2,1\}$$

d)

$$\int \frac{dx}{(x+2)(x^2+4x+4)} = \int \frac{dx}{(x+2)^3} = -\frac{1}{2(x+2)^2}+C, x\neq\{-2\}$$

e)

$$\int \frac{3x-2}{x^4-x^3}dx = \int \frac{3x-2}{x^3(x-1)}dx = \int \frac{2}{x^3}-\frac{1}{x^2}-\frac{1}{x}+\frac{1}{(x-1)}dx = \frac{2}{-2x^2}-\frac{1}{-1x}-\ln|x|+\ln|x-1|+C =$$

$$-\frac{1}{x^2}+\frac{1}{x}+\ln\left|\frac{x-1}{x}\right|+C, x\neq\{0,1\}$$

f)

$$\int \frac{x^4+x^3+11x^2-7x}{(x+1)^3(x^2-4x+4)}dx = \int \frac{x^4+x^3+11x^2-7x}{(x+1)^2(x-2)^2}dx = \int \frac{2}{(x+1)^3}-\frac{2}{(x+1)^2}+\frac{2}{(x-2)^2}+\frac{1}{(x-2)}dx =$$

$$= \frac{2}{-2(x+1)^2}-\frac{2}{-1(x+1)}+\frac{2}{-1(x-2)}+\ln|x-2|+C = -\frac{1}{(x+1)^2}+\frac{2}{(x+1)}-\frac{2}{(x-2)}+\ln|x-2|+C, x\neq\{-1,2\}$$

g)

$$\int \frac{-5}{(x+4)^4}dx = -5\int \frac{1}{(x+4)^4}dx = \frac{-5}{-3(x+4)^3}+C = \frac{5}{3(x+4)^3}+C, x\neq\{-4\}$$

h)

$$\int \frac{3}{(2x-1)^3}dx = \frac{3}{2}\frac{1}{-2(2x-1)^2}+C = -\frac{3}{4(2x-1)^2}+C, x\neq\left\{-\frac{1}{2}\right\}$$

str. 126 př. 2 (1.derivace čitatele + substituce, parciální zlomky)

a)

$$\int \frac{x+3}{x^2+2x+10}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+10}dx + \int \frac{2}{x^2+2x+10}dx$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x+10}dx = \frac{1}{2}\ln\left|x^2+2x+10\right| + C$$

$$\int \frac{2}{x^2+2x+10}dx = 2\int \frac{1}{\left(\frac{x+1}{3}\right)^2+1}dx = \left| \begin{matrix} \left(\frac{x+1}{3}\right)=t \\ \frac{1}{3}dx = dt \end{matrix} \right| = 2\int \frac{1}{3}\frac{1}{t^2+1}dt = \frac{2}{3}\arctan(t) + C = \frac{2}{3}\arctan\left(\frac{x+1}{3}\right) + C$$

$$\int \frac{x+3}{x^2+2x+10}dx = \frac{1}{2}\ln\left|x^2+2x+10\right| + \frac{2}{3}\arctan\left(\frac{x+1}{3}\right) + C, x \in R$$

b)

$$\int \frac{5x-2}{x^2-2x+5}dx = \frac{5}{2}\int \frac{2x-2}{x^2-2x+5}dx + \int \frac{3}{x^2-2x+5}dx$$

$$\frac{5}{2}\int \frac{2x-2}{x^2-2x+5}dx = \frac{5}{2}\ln\left|x^2-2x+5\right| + C$$

$$\int \frac{3}{x^2-2x+5}dx = 3\int \frac{1}{4\left(\left(\frac{x-1}{2}\right)^2+1\right)}dx = \left| \begin{matrix} \left(\frac{x-1}{2}\right)=t \\ \frac{1}{2}dx = dt \end{matrix} \right| = 3\int \frac{2}{4\left(t^2+1\right)}dt = \frac{3}{2}\arctan(t) + C = \frac{3}{2}\arctan\left(\frac{x-1}{2}\right) + C$$

$$\int \frac{5x-2}{x^2-2x+5}dx = \frac{5}{2}\ln\left|x^2-2x+5\right| + \frac{3}{2}\arctan\left(\frac{x-1}{2}\right) + C, x \in R$$

c)

$$\int \frac{3x+4}{x^2+4x+13}dx = \frac{3}{2}\int \frac{2x+4}{x^2+4x+13}dx - \int \frac{2}{x^2+4x+13}dx$$

$$\frac{3}{2}\int \frac{2x+4}{x^2+4x+13}dx = \frac{3}{2}\ln\left|x^2+4x+13\right| + C$$

$$-\int \frac{2}{x^2+4x+13}dx = -2\int \frac{1}{\left(\frac{x+2}{3}\right)^2+1}dx = \left| \begin{matrix} \left(\frac{x+2}{3}\right)=t \\ \frac{1}{3}dx = dt \end{matrix} \right| = -2\int \frac{1}{3\left(t^2+1\right)}dt = -\frac{2}{3}\arctan(t) + C = -\frac{2}{3}\arctan\left(\frac{x+2}{3}\right) + C$$

$$\int \frac{3x+4}{x^2+4x+13}dx = \frac{3}{2}\ln\left|x^2+4x+13\right| - \frac{2}{3}\arctan\left(\frac{x+2}{3}\right) + C, x \in R$$

d)

$$\int \frac{x}{x^2-6x+13}dx = \frac{1}{2}\int \frac{2x-6}{x^2-6x+13}dx + \int \frac{3}{x^2-6x+13}dx$$

$$\frac{1}{2}\int \frac{2x-6}{x^2-6x+13}dx = \frac{1}{2}\left|\ln x^2-6x+13\right| + C$$

$$\int \frac{3}{x^2-6x+13}dx = 3\int \frac{1}{4\left(\left(\frac{x-3}{2}\right)^2+1\right)}dx = \left| \begin{matrix} \left(\frac{x-3}{2}\right)=t \\ dx = 2dt \end{matrix} \right| = 3\int \frac{2}{4\left(t^2+1\right)}dt = \frac{3}{2}\arctan(t) + C = \frac{3}{2}\arctan\left(\frac{x-3}{2}\right) + C$$

$$\int \frac{x}{x^2-6x+13}dx = \frac{1}{2}\left|\ln x^2-6x+13\right| + \frac{3}{2}\arctan\left(\frac{x-3}{2}\right) + C, x \in R$$

e)

$$\int \frac{dx}{4x^2-12x+13}dx = \int \frac{1}{4\left(\left(\frac{2x-3}{2}\right)^2+1\right)}dx = \left| \begin{matrix} \left(\frac{2x-3}{2}\right)=t \\ dx=dt \end{matrix} \right| = \int \frac{1}{4\left(t^2+1\right)}dt = \frac{1}{4}\arctan\left(t\right)+C =$$

$$= \frac{1}{4}\arctan\left(\frac{2x-3}{2}\right)+C, x \in R$$

f)

$$\int \frac{dx}{x\left(x^2+1\right)^2} = \int \frac{1}{x}dx - \int \frac{x}{x^2+1}dx = \int \frac{1}{x}dx - \frac{1}{2}\int \frac{2x}{x^2+1}dx = \ln|x| - \frac{1}{2}\ln\left|x^2+1\right| + C = \frac{1}{2}\ln\frac{x^2}{x^2+1}+C, x \neq 0$$

g)

$$\int \frac{dx}{\left(x+1\right)^2\left(x^2+1\right)} = \int \frac{1}{2\left(x+1\right)^2}dx + \int \frac{1}{2\left(x+1\right)}dx - \frac{1}{4}\int \frac{2x}{\left(x^2+1\right)}dx =$$

$$= -\frac{1}{2\left(x+1\right)} + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left|x^2+1\right| + C = -\frac{1}{2\left(x+1\right)} + \frac{1}{4}\ln\frac{\left(x+1\right)^2}{\left|x^2+1\right|} + C, x \neq -1$$

h)

$$\int \frac{x}{\left(x^2+1\right)^3}dx = \frac{1}{2.(-2)}\left(x^2+1\right)^{-2} + C = -\frac{1}{4\left(x^2+1\right)^2} + C, x \in R$$

str. 126 př. 3 (exp)

a)

$$\int \frac{3e^{2x}}{e^{4x}+e^{2x}-2}dx = \left| \begin{matrix} e^{2x}=t \\ x=\ln t/2 \\ dx=dt/2t \end{matrix} \right| = \int \frac{3t}{t^2+t-2}.\frac{1}{2t}dt = \frac{3}{2}\int \frac{1}{\left(t-1\right)\left(t+2\right)}dt = \frac{3}{2}\int \frac{1}{3\left(t-1\right)} - \frac{1}{3\left(t+2\right)}dt =$$

$$= \frac{3}{2}\left(\frac{1}{3}\ln|t-1| - \frac{1}{3}\ln|t+2|\right) + C = \frac{1}{2}\ln\left|e^{2x}-1\right| - \frac{1}{2}\ln\left|e^{2x}+2\right| + C = \frac{1}{2}\ln\left|\frac{e^{2x}-1}{e^{2x}+2}\right| + C, x \neq 0$$

b)

$$\int \frac{2}{e^{3x}+2}dx = \left| \begin{matrix} e^{3x}=t \\ x=\ln t/3 \\ dx=dt/3t \end{matrix} \right| = \int \frac{2}{t+2}\frac{dt}{3t} = \frac{2}{3}\int \frac{1}{t.\left(t+2\right)}dt = \frac{2}{3}\int \frac{1}{2t} - \frac{1}{2\left(t+2\right)}dt = \frac{2}{3}\left(\frac{1}{2}\ln|t| - \frac{1}{2}\ln|t+2|\right) + C =$$

$$= \frac{1}{3}\ln e^{3x} - \frac{1}{3}\ln\left|e^{3x}+2\right| + C = \frac{3x}{3}\ln e - \frac{1}{3}\left|e^{3x}+2\right| + C = x - \frac{1}{3}\ln\left|e^{3x}+2\right| + C, x \in R$$

c)

$$\int \frac{e^{2x}}{e^{4x}-2e^{2x}+2}dx = \left| \begin{matrix} e^{2x}=t \\ x=\ln 2 \\ dx=dt/2t \end{matrix} \right| = \int \frac{t}{t^2-2t+2}\frac{dt}{2t} = \frac{1}{2}\int \frac{1}{\left(t^2-2t+1\right)+1}dt = \frac{1}{2}\int \frac{1}{\left(t-1\right)^2+1}dt =$$

$$= \frac{1}{2}\arctan\left(t-1\right) + C = \frac{1}{2}\arctan\left(e^{2x}-1\right) + C, x \in R$$

d)

$$\int \frac{dx}{e^x+e^{-x}} = \left| \begin{matrix} e^x=t \\ dx=dt \end{matrix} \right| = \int \frac{1}{t+\frac{1}{t}}dt = \int \frac{1}{t^2+1}dt = \arctan\left(t\right) + C = \arctan\left(e^x\right) + C, x \in R$$

e)

$$\int \frac{\ln x}{x\left(\ln^2 x-4\right)}dx=\left|\begin{matrix}\ln x=t\\ x=e^t\\ dx/x=dt\end{matrix}\right|=\int\frac{t}{t^2-4}dt=\int\frac{1}{(t-2)(t+2)}dt=\int\frac{1}{2(t-2)}+\frac{1}{2(t+2)}dt=$$

$$\frac{1}{2}\left(\ln|t-2|+\ln|t+2|\right)+C=\frac{1}{2}\ln\left(t^2-4\right)+C=\frac{1}{2}\ln\left(\ln^2 x-4\right)+C,x\neq\left\{e^{\pm 2}\right\}$$

f)

$$\int\frac{dx}{x\ln x}=\left|\begin{matrix}t=\ln x\\ dx/x=dt\end{matrix}\right|=\int\frac{1}{t}dt=\ln|t|+C=\ln|\ln x|+C,x\in(0,1)\bigcap(1,\infty)$$

str. 126 př. 4

a)

$$\int\sin^6(x).\cos^3(x)dx=\int\sin^6(x).\cos^2(x).\cos(x)dx=\int\sin^6(x).\left(1-\sin^2(x)\right)\cos(x)dx=\left|\begin{matrix}t=\cos x\\ \cos(x)dx=dt\end{matrix}\right|=$$

$$=\int t^6\left(1-t^2\right)dt=\int t^6-t^8dt=\frac{1}{7}t^7-\frac{1}{9}t^9+C=\frac{1}{7}\cos^7(x)-\frac{1}{9}\cos^9(x)+C,x\in R$$

b)

$$\int\sin^3(x).\cos^5(x)dx=\int\sin^2(x).\cos^5(x).\sin(x)dx=\left|\begin{matrix}t=\cos(x)\\ -\sin(x)dx=dt\end{matrix}\right|=-\int\left(1-t^2\right)t^5dt=-\int t^5-t^7dt=$$

$$=-\frac{1}{6}t^6+\frac{1}{8}t^8+C=-\frac{1}{6}\cos^6(x)+\frac{1}{8}\cos^8(x)+C,x\in R$$

c)

$$\int\frac{\cos x}{cos^2x-\sin x+1}dx=\int\frac{1}{\left(1-\sin^2 x\right)-\sin x+1}\cos(x)dx=\left|\begin{matrix}t=\sin x\\ \cos(x)dx=dt\end{matrix}\right|=\int\frac{1}{1-t^2-t+1}dt=$$

$$=\int\frac{-1}{(t+2)(t-1)}dt=\int\frac{1}{3(t+2)}-\frac{1}{3(1-t)}dt=\frac{1}{3}\ln|t+2|-\frac{1}{3}\ln|1-t|+C=$$

$$=\frac{1}{3}\ln\frac{|\sin x+2|}{|1-\sin x|}+C,x\in\left(-\frac{3}{2}\pi+2k\pi,\frac{1}{2}\pi+2k\pi\right),k\in Z$$

d)

$$\int\frac{1}{(1+\cos x)^2}dx=\begin{matrix}1+\cos x=t\\ \sin xdx=-dt\end{matrix}=-\int\frac{1}{t^2}dt=-\left(-\frac{1}{t}\right)+C=\frac{1}{t}+C=\frac{1}{1+\cos x}+C,x\in(\pi+k\pi)k\in Z$$

e)

$$\int\tan^4(x)dx=\frac{1}{1-4}\tan^3(x)-\int\tan^2(x)dx=-\frac{1}{3}\tan^3(x)-\frac{1}{1-2}\tan x-\int 1dx=$$

$$-\frac{1}{3}\tan^3(x)+\frac{1}{2}\tan x-x+C,x\in\left(\pm\frac{\pi}{2}+k\pi\right),k\in Z$$

f)

$$\int\frac{1-\sin x}{1+\cos x}dx=\int\frac{1-\frac{2tg\left(x/2\right)}{1+tg^2\left(x/2\right)}}{1+\frac{1-tg^2\left(x/2\right)}{1+tg^2\left(x/2\right)}}=\left|\begin{array}{c}t=tg\left(x/2\right)\\x=2\arctan\left(t\right)\\dx=\frac{2}{1+t^2}dt\end{array}\right|=\int\frac{1-\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}\frac{2}{1+t^2}dt=\int\frac{1+t^2-2t}{1+t^2}.\frac{1+t^2}{1+t^2+1-t^2}.\frac{2}{1+t^2}dt=$$

$$\int\frac{t^2-2t+1}{t^2+1}dt=\int\frac{t^2}{t^2+1}dt-\int\frac{2t}{t^2+1}dt+\int\frac{1}{t^2+1}dt=\int 1dt-\int\frac{1}{t^2+1}dt-\int\frac{2t}{t^2+1}dt+\int\frac{1}{t^2+1}dt=$$

$$t-\arctan\left(t\right)-\ln\left(t^2+1\right)+\arctan\left(t\right)+C=tg\left(\frac{x}{2}\right)-\ln\left(tg^2\left(\frac{x}{2}\right)+1\right)+C,x\in\left(\pm\pi+2k\pi\right),k\in Z$$

g)

$$\int\operatorname{cotan}^2\left(x\right)dx=-\frac{1}{1}\operatorname{cotan}\left(x\right)-\int 1dx=-\operatorname{cotan}\left(x\right)-x+C,x\in\left(k\pi\right),k\in Z$$

h)

$$\int\frac{dx}{1-\cos x}=\int\frac{dx}{1-\frac{1-\tan^2\left(x/2\right)}{1+\tan^2\left(x/2\right)}}=\left|\begin{array}{c}t=\tan\left(x/2\right)\\x=2\arctan\left(t\right)\\dx=\frac{2}{t^2+1}dt\end{array}\right|=\int\frac{1}{1-\left(\frac{1-t^2}{1+t^2}\right)}.\frac{2}{t^2+1}dt=\int\frac{t^2+1}{1+t^2-1+t^2}.\frac{2}{t^2+1}dt=$$

$$=\int\frac{1}{t^2}dt=-\frac{1}{t}+C=-\frac{1}{\tan\left(x/2\right)}=-\cot\tan\left(\frac{x}{2}\right)+C,x\in\left(0+2k\pi\right),k\in Z$$

str. 126 př. 5

a)

$$\int x.\sqrt[3]{x-1}=\left|\begin{array}{c}t^3=x-1\\x=t^3+1\\dx=3t^2dt\end{array}\right|=\int\left(t^3+1\right)t3t^2dt=\int\left(t^3+1\right)3t^3dt=\int 3t^6+3t^3dt=\frac{3}{7}t^7+\frac{3}{4}t^4+C=$$

$$\frac{3}{7}\sqrt[3]{\left(x-1\right)^7}+\frac{3}{4}\sqrt[4]{\left(x-1\right)^4}+C,x\in R$$

b)

$$\int\frac{x-1}{\sqrt{2x+1}}dx=\left|\begin{array}{c}t^2=2x+1\\x=\frac{t^2-1}{2}\\dx=tdt\end{array}\right|=\frac{\frac{t^2-1}{2}-\frac{2}{2}}{t}tdt=\frac{t^2-3}{2}dt=\frac{1}{2}\int t^2-\frac{3}{2}\int 1dt=\frac{1}{6}t^3-\frac{3}{2}t+C=$$

$$=\frac{1}{6}\sqrt{\left(2x+1\right)^3}-\frac{3}{2}\sqrt{2x+1}+C,x\in\left(-\frac{1}{2},+\infty\right)$$

c)

$$\int\frac{dx}{2+\sqrt{x+1}}=\left|\begin{array}{c}t^2=x+1\\x=t^2-1\\dx=2tdt\end{array}\right|=2\int\frac{t}{2+t}dt=2\int 1-\frac{2}{t}dt=2t-4\ln\left|t\right|+C=2\sqrt{x+1}-4\ln\sqrt{x+1}+C,x\in\left(-1,+\infty\right)$$

d)

$$\int \frac{dx}{x\sqrt{x+1}} = \begin{vmatrix} t^2 = x+1 \\ x = t^2 - 1 \\ dx = 2tdx \end{vmatrix} = \int \frac{2t}{\left(t^2-1\right)t}dt = \int \frac{2}{t^2-1} = \int \frac{1}{t-1} - \frac{1}{t+1}dt = \ln\left|\frac{t-1}{t+1}\right| + C =$$

$$\ln\frac{\left|\sqrt{x+1}-1\right|}{\sqrt{x+1}+1} + C, x \in \left(0, +\infty\right)$$

e)

$$\int \frac{\sqrt{x-4}}{x}dx = \begin{vmatrix} t^2 = x-4 \\ x = t^2+4 \\ dx = 2tdt \end{vmatrix} = \int \frac{t.2t}{t^2+4}dt = \int \frac{2t^2}{t^2+4}dt = \int \frac{2.4.\frac{t^2}{4}}{4.\left(\frac{t^2}{4}+1\right)}dt = \int \frac{8\left(\frac{t}{2}\right)^2}{4\left(\left(\frac{t}{2}\right)^2+1\right)}dt = 2\int \frac{\left(\frac{t}{2}\right)^2}{\left(\frac{t}{2}\right)^2+1}dt =$$

$$= 2\int 1 - \frac{1}{\left(\frac{t}{2}\right)^2+1}dt = 2t - \frac{2}{\frac{1}{2}}\operatorname{arctan}\left(\frac{t}{2}\right) + C = 2\sqrt{x-4} - 4\operatorname{arctan}\left(\frac{\sqrt{x-4}}{2}\right) + C, x \in \left(4, +\infty\right)$$

f)

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{dx}{x^{\frac{3}{6}}+x^{\frac{2}{6}}} = \begin{vmatrix} t^6 = x \\ t = \sqrt[6]{x} \\ dx = 6t^5dt \end{vmatrix} = \int \frac{6t^5}{t^3+t^2}dt = 6\int t^2 - t + 1 - \frac{1}{t+1}dt =$$

$$\frac{6}{3}t^3 - \frac{6}{2}t^2 + t - 6\ln\left|t+1\right| + C = 2x^{\frac{3}{6}} - 3x^{\frac{2}{6}} + x^{\frac{1}{6}} - 6\ln\left|t+1\right| + C = 2\sqrt{x} - 3\sqrt[3]{x} + \sqrt[6]{x} - 6\ln\left|t+1\right| + C, x > 0$$

str. 126 př. 6

a)

$$\int \sqrt{-x^2+6x-8}dx = \int \sqrt{-x^2+6x-9+9-8}dx = \int \sqrt{1-\left(x-3\right)^2}dx = \begin{vmatrix} \left(x-3\right) = t \\ dx = dt \end{vmatrix} = \int \sqrt{1-t^2}dt =$$

$$= \begin{vmatrix} t = \sin y \\ dt = \cos\left(y\right)dy \\ y = \arcsin t \end{vmatrix} = \int \sqrt{1-\sin^2 y}.\cos\left(y\right)dy = \int \left|\cos y\right|.\cos\left(y\right)dy = \int \cos^2\left(y\right)dy =$$

$$= \frac{1}{2}\int 1 + \cos\left(2y\right)dy = \frac{1}{2}y + \frac{1}{2}.\frac{1}{2}\sin\left(2y\right) + C = \frac{1}{2}y + \frac{1}{4}2\sin y.\cos y + C = \frac{1}{2}y + \frac{1}{2}\sin y.\sqrt{1-\sin^2 y} + C =$$

$$= \frac{1}{2}\arcsin\left(t\right) + \frac{1}{2}t.\sqrt{1-t^2} + C = \frac{1}{2}\arcsin\left(x-3\right) + \frac{1}{2}\left(x-3\right).\sqrt{1-\left(x-3\right)^2} + C =$$

$$= \frac{1}{2}\arcsin\left(x-3\right) + \frac{1}{2}\left(x-3\right).\sqrt{-x^2+6x-8} + C, x \in \left(2, 4\right)$$

b)

$\arg\cosh x = \ln\left(x + \sqrt{x^2 - 1}\right)$

$$\int \frac{dx}{\sqrt{x^2+4x}} = \int \frac{dx}{\sqrt{x^2+4x+4-4}} = \int \frac{dx}{\sqrt{(x+2)^2-4}} = \left| \begin{array}{c} (x+2) = 2\cosh t \\ t = \arg\cosh\left(\dfrac{x+2}{2}\right) \\ dx = 2\sinh t \end{array} \right| = \int \frac{2\sinh t dt}{\sqrt{4\cosh^2 t - 4}} =$$

$$\int \frac{2\sinh t}{2\sqrt{\cosh^2 t - 1}} dt = \int \frac{\sinh t}{\sinh t} dt = \int 1 dt = t + C = \arg\cosh\left(\frac{x+2}{2}\right) + C =$$

$$= \ln\left(\frac{x+2}{2} + \sqrt{\frac{x^2+4x+4}{4} - 1}\right) + C = \ln\left(x+2+\sqrt{x^2+4}\right) - \ln 2 + C = \ln\left(x+2+\sqrt{x^2+4}\right) + C, x \in (0, +\infty)$$

c)

$\arg\sinh x = \ln\left(x + \sqrt{x^2 + 1}\right)$

$$\int \frac{dx}{\sqrt{x^2-2x+10}} = \int \frac{dx}{\sqrt{x^2-2x+1+9}} = \int \frac{dx}{\sqrt{(x-1)^2+9}} = \left| \begin{array}{c} (x-1) = 3\sinh t \\ t = \arg\sinh\left(\dfrac{x-1}{3}\right) \\ dx = 3\cosh t dt \end{array} \right| = \int \frac{3\cosh t}{\sqrt{9\sinh^2 t + 9}} dt =$$

$$\int \frac{3\cosh t}{3\sqrt{\sinh^2 t + 1}} dt = \int \frac{\cosh t}{\cosh t} dt = \int 1 dt = t + C = \arg\sinh\left(\frac{x-1}{3}\right) + C = \ln\left(\frac{x-1}{3} + \sqrt{\frac{x^2-2x-1}{9} + 1}\right) =$$

$$= \ln\left(x-1+\sqrt{x^2-2x+10}\right) - \ln 2 + C = \ln\left(x-1+\sqrt{x^2-2x+10}\right) + C, x \in R$$