str. 59, př. 1

$$\lim_{x\to -1}(2x+5)=2.(-1)+5=3$$

$$\lim_{x\to -\infty}(3x^2+1)=\infty$$

$$\lim_{x\to 1+}\frac{3x^2+5}{x-1}=\lim_{x\to 1+}\frac{x^2}{x}\frac{3+\frac{5}{x^2}}{1-\frac{1}{x}}=1.\frac{3+\frac{5}{0+}}{1-\frac{1}{1+}}=\infty$$

$$\lim_{x\to\infty}\frac{2x-1}{x+4}=\lim_{x\to\infty}\frac{2-\frac{1}{x}}{1+\frac{4}{x}}=\frac{2-0}{1+0}=2$$

$$\lim_{x\to\infty} sign\left|\sin x\right|=\lim_{x\to\infty}1\Rightarrow \textit{neexistuje}$$

str. 59, př. 2

$$\lim_{x\to 0}\frac{\sin^2 x}{x}=\lim_{x\to 0}\frac{\sin x.\sin x}{x}=1.0=0$$

$$\lim_{x\to 0}\frac{\tan x}{2x}=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2x}=\lim_{x\to 0}\frac{\sin x}{2x.\cos x}=\lim_{x\to 0}\frac{1}{2\cos x}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{2^x-1}{\ln(x+1)}=\lim_{x\to 0}\frac{\frac{2^x-1}{x}}{\frac{\ln(x+1)}{x}}=\lim_{x\to 0}\frac{2^x-1}{1}=\lim_{x\to 0}x.\ln 2-\ln 1=\ln 2$$

str. 59, př. 3

$$\lim_{x\to\infty}\frac{x^2+2x-1}{x^3+3}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}-\frac{1}{x^3}}{1+\frac{3}{x^3}}=\frac{0+0-0}{1+0}=0$$

$$\lim_{x\to\infty}\frac{2x^3-1}{x^4+1}=\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^4}}{1+\frac{1}{x^4}}=\frac{0-0}{1+0}=0$$

$$\lim_{x\to\infty}\frac{4x^2-2x+1}{2x^2+3x-1}=\lim_{x\to\infty}\frac{4-\frac{2}{x}+\frac{1}{x^2}}{2+\frac{3}{x}+-\frac{1}{x^2}}=\frac{4-0+0}{2+0-0}=2$$

$$\lim_{x\to -\infty}\frac{3x^3-2x+5}{2x^2+3x-1}=\lim_{x\to\infty}\frac{3-\frac{2}{x^2}+\frac{5}{x^3}}{1+\frac{1}{x}+\frac{1}{x^3}}=\frac{3-0+0}{1+0+0}=3$$

$$\lim_{x\to\infty}\frac{x^3+2x}{x^3+x^2+1}=\lim_{x\to\infty}x^2.\left(\frac{x+\frac{2}{x}}{1+\frac{1}{x^2}}\right)=\infty.\left(\frac{\infty+\frac{2}{\infty}}{1+\frac{1}{\infty^2}}\right)=\infty$$

$$\lim_{x\to -\infty}\frac{2x^3+x^2+1}{x^2+x+3}=\lim_{x\to -\infty}x^3.\frac{2+\frac{1}{x}+\frac{1}{x^3}}{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}=-\infty.\frac{2+0+0}{0+0+0}=-\infty$$

$$\lim_{x\to-\infty}\frac{x^5+x^3+2}{2x^3-1}=\lim_{x\to-\infty}\frac{x^5}{x^3}.\frac{1+\frac{1}{x^2}+\frac{2}{x^5}}{2-\frac{1}{x^3}}=\infty.\frac{1+0+0}{2-0}=\infty$$

$$\lim_{x\to-\infty}\frac{1-x^5}{1+x^2}=\lim_{x\to-\infty}\frac{x^5}{x^2}.\frac{-\frac{1}{x^5}+1}{\frac{1}{x^2}+1}=-\infty.\frac{0+1}{0+1}=-\infty$$

str. 59, př. 4

$$\lim_{x\to-1}\frac{x^2-2x-1}{x^2+2x-2}=\lim_{x\to-1}\frac{1-\frac{2}{x}-\frac{1}{x^2}}{1+\frac{2}{x}-\frac{2}{x^2}}=\frac{1+2-1}{1-2-2}=-\frac{2}{3}$$

$$\lim_{x\to2}\frac{x^2-3x+2}{x^2+1}=\frac{4-6+2}{4+1}=\frac{0}{5}=0$$

$$\lim_{x\to2}\frac{x^2-x-2}{x^2-3x+2}=\lim_{x\to2}\frac{(1+x).(x-2)}{(x-1).(x-2)}=\lim_{x\to2}\frac{1+x}{x-1}=3$$

$$\lim_{x\to2}\frac{x+3}{x^2-x-2}=\lim_{x\to2}\frac{\frac{1}{x}+\frac{3}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}=\frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}-\frac{2}{4}}=\frac{\frac{5}{4}}{0}\Rightarrow \textit{neexistuje}$$

$$\lim_{x\to3}\frac{x-4}{x^2-4x+3}=\lim_{x\to3}\frac{\frac{1}{x}-\frac{4}{x^2}}{1-\frac{4}{x}+\frac{3}{x^2}}=\frac{\frac{1}{3}-\frac{4}{9}}{1-\frac{4}{3}+\frac{3}{9}}=\frac{-\frac{1}{9}}{0}\Rightarrow \textit{neexistuje}$$

$$\lim_{x\to-1}\frac{x+2}{x^2+2x+1}=\lim_{x\to-1}\frac{1}{0}=\infty$$

$$\lim_{x\to1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)=\lim_{x\to1}\frac{2(x-1)-(x^2-1)}{(x-1)^2.(x+1)}=\lim_{x\to1}\frac{-x^2+2x-1}{(x-1)^2.(x+1)}=\lim_{x\to1}\frac{-1.(x-1)^2}{(x+1).(x-1)^2}=-\frac{1}{2}$$

str. 59, př. 5

a)

$$\lim_{x\to\infty}\frac{x+1}{\sqrt{x-1}+2}=\lim_{x\to\infty}\frac{(x+1).\left(\sqrt{x-1}-2\right)}{\left(\sqrt{x-1}+2\right)\left(\sqrt{x-1}-2\right)}=\lim_{x\to\infty}\frac{x\sqrt{x-1}-2x+\sqrt{x-1}-2}{x-1-4}=$$

$$\lim_{x\to\infty}\frac{\sqrt{x-1}-2+\sqrt{\frac{1}{x}-\frac{1}{x^2}}+\frac{2}{x}}{1-\frac{1}{x}-\frac{4}{x}}=\lim_{x\to\infty}\frac{\infty-2+0-0}{1-0-0}=\infty$$

b)

$$\lim_{x\to\infty}\frac{3x-2}{\sqrt{x^2-4}-2}=\lim_{x\to\infty}\frac{3-\frac{2}{x}}{\sqrt{1-\frac{4}{x^2}}-\frac{2}{x}}=\frac{3-0}{\sqrt{1-0}-0}=3$$

c)

$$\lim_{x\to-\infty}\frac{2x+1}{\sqrt{4x^2-2}+1}=-\lim_{x\to\infty}\frac{2+\frac{1}{x}}{\sqrt{4-\frac{2}{x^2}}+\frac{1}{x}}=\frac{-2-0}{2}=-1$$

d)

$$\lim_{x\to6}\frac{\sqrt{x-2}-2}{x-6}=\lim_{x\to6}\frac{\left(\sqrt{x-2}-2\right).\left(\sqrt{x-2}+2\right)}{(x-6)\left(\sqrt{x-2}+2\right)}=\lim_{x\to6}\frac{x-2-4}{(x-6)\left(\sqrt{x-2}+2\right)}=\frac{1}{\sqrt{4}+2}=\frac{1}{4}$$

e)

$$\lim_{x\to-1}\frac{x+1}{\sqrt{x+2}-1}=\lim_{x\to-1}\frac{x+1}{\sqrt{x+2}-1}.\frac{\sqrt{x+2}+1}{\sqrt{x+2}+1}=\lim_{x\to-1}\frac{(x+1)\left(\sqrt{x+2}+1\right)}{x+2-1}=\sqrt{1}+1=2$$

$$\lim_{x\to\infty}\frac{\sqrt{x-1}+\sqrt{x+1}}{x}=\lim_{x\to\infty}\frac{x-1-(x+1)}{x\left(\sqrt{x-1}-\sqrt{x+1}\right)}=\lim_{x\to\infty}\frac{-2}{\infty}=0$$

f)

$$\lim_{x\to\infty}\frac{\sqrt{x-1}+\sqrt{x+1}}{x}=\lim_{x\to\infty}\frac{x-1-(x+1)}{x\left(\sqrt{x-1}-\sqrt{x+1}\right)}=\lim_{x\to\infty}\frac{-2}{\infty}=0$$

g)

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)=\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right).\frac{\left(\sqrt{x^2+2x}+x\right)}{\left(\sqrt{x^2+2x}+x\right)}=\lim_{x\to\infty}\frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x}=\lim_{x\to\infty}\frac{2}{\sqrt{x^2+2x}+x}=$$

$$\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{2}{x}}+1}=\frac{2}{\sqrt{1+0}+1}=\frac{2}{2}=1$$

h)

$$\lim_{x\to\infty}\left(\sqrt{x^2+2}+x\right)=\lim_{x\to\infty}\left(\sqrt{x^2+2}+x\right).\frac{\left(\sqrt{x^2+2}-x\right)}{\left(\sqrt{x^2+2}-x\right)}=\lim_{x\to\infty}\frac{x^2+2-x^2}{\sqrt{x^2+2}-x}=\lim_{x\to\infty}\frac{2}{\sqrt{x^2+2x}+x}=$$

$$\lim_{x\to\infty}\frac{\frac{2}{x}}{\sqrt{1+\frac{2}{x}}+1}=\frac{0}{\sqrt{1+0}+1}=0$$

i)

$$\lim_{x\to-\infty}\left(\sqrt{x^2+2}-x\right)=\sqrt{\infty+2}+\infty=\infty$$

str. 59, př. 6

a)

$$\lim_{x\to\infty}\frac{x+\sin(x)}{x+\cos(x)}=\lim_{x\to\infty}\frac{\frac{x+\sin(x)}{x}}{\frac{x+\cos(x)}{x}}=\lim_{x\to\infty}\frac{\frac{x}{x}+0}{\frac{x}{x}+0}=\frac{1}{1}=1$$

b)

$\lim\limits_{x\to 1}\frac{1}{\sqrt{1-x^2}}=$ neexistuje, protože v $x=1$ v není spojitá.

c)

d)

$$\lim_{x\to-\infty}\frac{\arctan(x)}{x}=\frac{-\frac{\pi}{2}}{-\infty}=0$$

$$\lim_{x\to\infty}\left(\ln x+\cos(x)\right)=$$

$$\lim_{x\to-\infty}\frac{x^2+x\sin(x)}{x+1}=\lim_{x\to-\infty}\frac{x+x\frac{\sin(x)}{x}}{\frac{x+1}{x}}=\lim_{x\to-\infty}\frac{x\left(x+x\frac{\sin(x)}{x}\right)}{x+1}=\lim_{x\to-\infty}\frac{x(x+0)}{x+1}=\lim_{x\to-\infty}\frac{x^2}{x+1}\lim_{x\to-\infty}\frac{2x}{1}=-\infty$$

str. 59, př. 7

a)

$$\lim_{x\to-\infty}\sqrt[3]{2x^2+3x-5}=\lim_{x\to-\infty}x^6\sqrt[3]{2+\frac{3}{x}-\frac{5}{x^2}}=(-\infty)^6+\sqrt[3]{2}=\infty$$

b)

$\lim\limits_{x\to 1}\sqrt{\frac{x+1}{x-1}}=$ neexistuje, funkce není spojitá v $x=1$.

c)

$\lim\limits_{x\to 0}\sin\left(\frac{1}{x}\right)=$ neexistuje, funkce není spojitá v $x=0$.

d)

$$\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)=\cos 0=1$$

e)

$$\lim_{x\to\infty}\ln^2(1+\cos x)=\lim_{x\to\infty}\ln(1+\cos x).\ln(1+\cos x)=(-\infty).(-\infty)=\infty$$

f)

$$\lim_{x\to\infty}xe^{\frac{1}{x}}=\infty.e^{\frac{1}{\infty}}=\infty.1=\infty$$

g)

$$\lim_{x\to\infty}\frac{3e^{2x}}{e^{2x}+1}=\lim_{x\to\infty}\frac{e^{2x}.3}{e^{2x}+e^{-2x}}=\lim_{x\to\infty}\frac{3}{1+e^{-2x}}=\frac{3}{1+0}=3$$

h)

$$\lim_{x\to\infty}\arcsin\left(\frac{1-x}{1+x}\right)=$$

DU

Určete rovnice tečen kolmých k $p: 2x+4y-3=0$ hyperboly $7x^2-2y^2=14$.

$$2x+4-3=0$$
$$y=-\frac{1}{2}x+\frac{3}{4}$$

$$k=-\frac{1}{2}$$
$$k_p.k_t=-1$$
$$k_t=2$$

$$7x^2-2y^2=14$$
$$y=\sqrt{\frac{7x^2-14}{2}}$$
$$y'=\frac{1}{2}\left(\frac{7x^2-14}{2}\right)^{-\frac{1}{2}}7x$$

$$\frac{7x}{\frac{1}{2}\left(\frac{7x^2-14}{2}\right)^{\frac{1}{2}}}=2$$
$$7x=4\left(\frac{7x^2-14}{2}\right)^{\frac{1}{2}}$$
$$\frac{7}{4}x=\sqrt{\frac{7x^2-14}{2}}$$
$$\frac{49}{16}x^2=\frac{7x^2-14}{2}$$
$$98x^2=112x^2-225$$
$$-14x^2+224=0$$
$$x_{1,2}=\pm 4$$

$$t: y=f(a)+f'(a)(x-a)$$
$$t_1: y=-1+2x$$
$$T_1=[4,7]$$
$$t_2: y=1+2x$$
$$T_2=[-4,-7]$$

Rovnice tečen jsou $t_1: y=-1+2x$ a $t_2: y=1+2x$ a body dotyku jsou $T_1=[4,7]$ a $T_2=[-4,-7]$.