str. 51, př. 1

Určete jaká je posloupnost $\frac{1}{n(n+1)}$.

$a_1 = \frac{1}{1.(1+1)} = \frac{1}{2}$ $\quad a_2 = \frac{1}{2.(2+1)} = \frac{1}{6}$ $\quad \Rightarrow \quad a_1 > a_2$

$$a_1 > a_2$$
$$\frac{1}{n(n+1)} > \frac{1}{(n+1).(n+1)}$$
$$\frac{1}{n^2+n} > \frac{1}{n^2+2n+1}$$
$$n^2+2n+1 > n^2+n \Rightarrow klesající$$

Posloupnost je klesající.

str. 51, př. 2

Určete a_{10}

$$a_2 + a_3 = 9 \qquad a_2 + a_3 = 9$$
$$a_2.a_3 = 14 \qquad a_3 = 9 - a_2$$

$$a_2.a_3 = 14$$
$$a_2.(9-a_2) = 14$$
$$9a_2 - a_2^2 = 14$$
$$a_2^2 - 9a_2^2 + 14 = 0$$
$$a_2 = \{2,7\}$$

$$a_{2a} + a_3 = 9$$
$$a_{2a} + a_{2a} + d = 9$$
$$d_{1,2} = 9 - 2a_{2a}$$
$$d_{1,2} = \{5,-5\}$$

$$a_{2a} + a_3 = 9$$
$$a_{2a} + a_{2a} + d = 9$$
$$d_{1,2} = 9 - 2a_{2a}$$
$$d_{1,2} = \{5,-5\}$$

$$a_1 = a_2 - d$$
$$a_1 = \{2,7\} - \{5,-5\}$$
$$a_1 = \{-3,12\}$$

$$a_{10} = a_1 + 9d$$
$$a_{10} = \{-3,12\} + 9.\{5,-5\}$$
$$a_{10} = \{42,-33\}$$

Pro $d = 5$ je $a_{10} = 42$ a pro $d = -5$ je $a_{10} = -33$.

str. 51, př. 3a

Určete a_1 a q

$$a_1 + a_4 = 195$$
$$a_2 + a_3 = 60$$

$$a_1 + a_1.q^3 = 195$$
$$a_1.(1+q^3) = 195$$
$$a_1 = \frac{195}{1+q^3}$$

$$a_2 + a_3 = 60$$
$$a_1q + a_1q^2 = 60$$
$$a_1q.(1+q) = 60$$

$$\frac{195}{1+q^3}q.(1+q)=60$$

$$\frac{195q}{q^2-q+1}=60$$

$$195q=60q^2-60q+60$$

$$4q^2-17q+4=0$$

$$q_{1,2}=\left\{\frac{1}{4},4\right\}$$

$$a_1+a_1.q^3=195$$

$$a_1=\frac{195}{1+q_1{}^3}=\frac{195}{1+\frac{1}{4^3}}=192$$

$$a_1=\frac{195}{1+q_2{}^3}=\frac{195}{1+4^3}=3$$

Pro $q_1=\frac{1}{4}$ je $a_1=192$ a $q_2=4$ je $a_1=3$

str. 51, př. 3b

Určete a_1 a q

$$a_1-a_2+a_3=15$$
$$a_4-a_5+a_6=120$$

$$a_1-a_2+a_3=15$$

$$a_1-a_1q+a_1q^2=15$$

$$a_1\left(q^2-q+1\right)=15$$

$$a_1=\frac{15}{q^2-q+1}$$

$$a_4-a_5+a_6=120$$

$$a_1q^3-a_1q^4+a_1q^5=120$$

$$a_1\left(q^5-q^4+q^3\right)=120$$

$$\frac{15\left(q^5-q^4+q^3\right)}{q^2-q+1}=120$$

$$15q^3=120$$

$$q=2$$

$$a_1=\frac{15}{q^2-q+1}$$

$$a_1=\frac{15}{4-2+1}=5$$

Posloupnost má řešení $q=2$ a $a_1=5$.

str. 51, př. 4

a)

$$\frac{5}{3}=x+3x^2+x^3+3x^4+\ldots$$

$$a_1=x+3x^2$$

$$a_2=x^3+3x^4$$

$$q=\frac{a_2}{a_1}=\frac{x^3+3x^4}{x+3x^2}=x^2$$

$$s_n=\frac{a_1}{1-q}=\frac{x+3x^2}{1-x^2}=-3+\frac{x+3}{1-x^2}$$

$\frac{5}{3} = s_n$

$\frac{5}{3} = -3 + \frac{x+3}{1-x^2}$

$\frac{x+3}{1-x^2} = \frac{5}{3} + \frac{9}{3}$

$3(x+3) = 14(1-x^2)$

$3x + 9 = -14x^2 + 14$

$14x^2 + 3x - 5 = 0$

$x_{1,2} = \left\{-\frac{5}{7}, \frac{1}{2}\right\}$

Řešení rovnice je $x_{1,2} = \left\{-\frac{5}{7}, \frac{1}{2}\right\}$.

b)

$\frac{5}{3} = x + 3x^2 + x^3 + 3x^4 + \dots$

$2^x + 4^x + 8^x + 16^x + \dots = 1$

$a_1 = 2^x$

$a_2 = 4^x$

$q = \frac{a_2}{a_1} = \frac{4^x}{2^x} = 2^x$

$s = \frac{a_1}{1-q} = \frac{2^x}{1-2^x}$

$s = 1$

$\frac{2^x}{1-2^x} = 1$

$2^x = 1 - 2^x$

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

Rovnice má jediné řešení $x = -1$.

c)

$\log x + \log\sqrt{x} + \log\sqrt[4]{x} + \dots = 2$

$a_1 = \log x$

$a_2 = \log\sqrt{x} = \log x^{\frac{1}{2}} = \frac{1}{2}\log x$

$q = \frac{a_2}{a_1} = \frac{\frac{1}{2}\log x}{\log x} = \frac{1}{2}$

$s = \frac{a_1}{1-q} = \frac{\log x}{1-\frac{1}{2}} = \frac{2\log x}{1}$

$s = 2$

$2\log x = 2$

$\log x = 1$

$x = 10$

Rovnice má jediné řešení $x = 10$.

d)

$$1+\frac{2}{x}+\frac{4}{x^2}+\frac{8}{x^3}+...=\frac{4x-3}{3x-4}$$

$a_1=1$

$a_2=\frac{2}{x}$

$q=\frac{a_2}{a_1}=\frac{2}{x}\Rightarrow x>2$

$s=\frac{a_1}{1-q}=\frac{1}{1-\frac{2}{x}}=\frac{x}{x-2}$

$s=\frac{4x-3}{3x-4}$

$\frac{x}{x-2}=\frac{4x-3}{3x-4}$

$x.(3x-4)=(4x-3).(x-2)$

$3x^2-4x=4x^2-11x+6$

$x^2-7x+6=0$

$x_{1,2}=\{6,1\}\Rightarrow x=6$

Rovnice má jediné řešení $x=6$.

str. 51, př. 6

Dokažte, že posloupnost $\frac{3n+4}{n}$ je roustoucí a má lim = 3.

$a_1=7$ $a_2=5$

$a_1>a_2$

$\frac{3n+4}{n}>\frac{3(n+1)+4}{n+1}$

$(3n+4).(n+1)>3n(n+1)+4n$

$3n^2+7n+4>3n^2+7n$

$\lim\limits_{n\to\infty}\frac{3n+4}{n}=\lim\limits_{n\to\infty}\frac{3}{1}=3$.

Posloupnost je skutečně rostoucí a má limitu v bodě $n=3$.

str. 51, př. 7

a)

$$\lim_{n\to\infty}\left(\frac{1+2+3+...n}{n+2}-\frac{n}{2}\right)=DODĚLAT$$

b)

$$\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{4}+\dots\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\dots+\frac{1}{3^n}}\right)=\lim_{n\to\infty}\frac{s_{n1}}{s_{n2}}=\frac{2}{\frac{3}{2}}=\frac{4}{3}$$

$$q_1=\frac{a_2}{a_1}=\frac{\frac{1}{2}}{1}=\frac{1}{2}$$

$$q_2=\frac{a_2}{a_1}=\frac{\frac{1}{3}}{1}=\frac{1}{3}$$

$$s_{n1}=\frac{a_1}{1-q_1}=\frac{1}{1-\frac{1}{2}}=2$$

$$s_{n2}=\frac{a_1}{1-q_2}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$$

c)

$$\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}=\lim_{n\to\infty}\frac{n!}{(n+1).n!-n!}=\lim_{n\to\infty}\frac{1}{(n+1).1-1}=\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0$$

str. 51, př. 8

a)

$$\frac{(n+1)!}{n!}-\frac{n!}{(n+1)!}=\frac{(n+1)n!}{n!}-\frac{n!}{(n+1)n!}=n+1-\frac{1}{n+1}=\frac{(n+1)^2-1}{n+1}=\frac{n(n+2)}{n+1}$$

b)

$$\frac{(n-1)!}{(n+1)!}+\frac{(3n+3)!}{(3n+4)!}=\frac{(n-1)!}{(n+1).n.(n-1)!}+\frac{(3n+3)!}{(3n+4).(3n+3)!}=\frac{1}{(n+1).n}+\frac{1}{(3n+4)}=$$

$$\frac{(3n+4)+(n+1).n}{n(n+1).(3n+4)}=\frac{n^2+4n+4}{n(n+1).(3n+4)}=\frac{(n+2)^2}{n(n+1).(3n+4)}$$

str. 51, př. 9

a)

$$\binom{x}{1}+\binom{x}{x-2}=66 \qquad x\ge 2$$

$$\frac{x!}{1!.(x-1)!}+\frac{x!}{(x-2)!.(x-x+2)!}=66$$

$$x+\frac{x.(x-1)}{2}=66$$

$$2x+x^2-x=132$$

$$x^2+x-132=0$$

$$x_{1,2}=\{-12,11\}$$

Řešení této rovnice je $x=11$.

b)

$$\binom{x-1}{x-3}+\binom{x-2}{x-4}=9 \quad x\geq 4$$

$$\frac{(x-1)!}{(x-3)!(x-1-x+3)!}+\frac{(x-2)!}{(x-4)!(x-2-x+4)!}=9$$

$$\frac{(x-1)(x-2)}{2}+\frac{(x-2)(x-3)}{2}=9$$

$$x^2-3x+2+x^2-5x+6=18$$

$$2x^2-8x+8=18$$

$$x^2-4x-5=0$$

$$x_{1,2}=\{-1,5\}$$

Řešení této rovnice je $x=5$.

str. 51, př. 10

Určete 10. člen binomického rozvoje $\left(\frac{\sqrt{x}}{x}+\sqrt[3]{x}\right)^{20}$.

$$A_{10}=\binom{20}{9}.\left(\frac{\sqrt{x}}{x}\right)^{11}.\left(\sqrt[3]{x}\right)^{9}=\binom{20}{9}.x^{\left(\frac{11}{2}-\frac{22}{2}\right)}.x^{3}=\binom{20}{9}.x^{\frac{-11}{2}+\frac{6}{2}}=\binom{20}{9}.x^{\frac{-5}{2}}=$$

$$=\binom{20}{9}x^{-3+\frac{1}{2}}=\binom{20}{9}\frac{\sqrt{x}}{x^3}$$