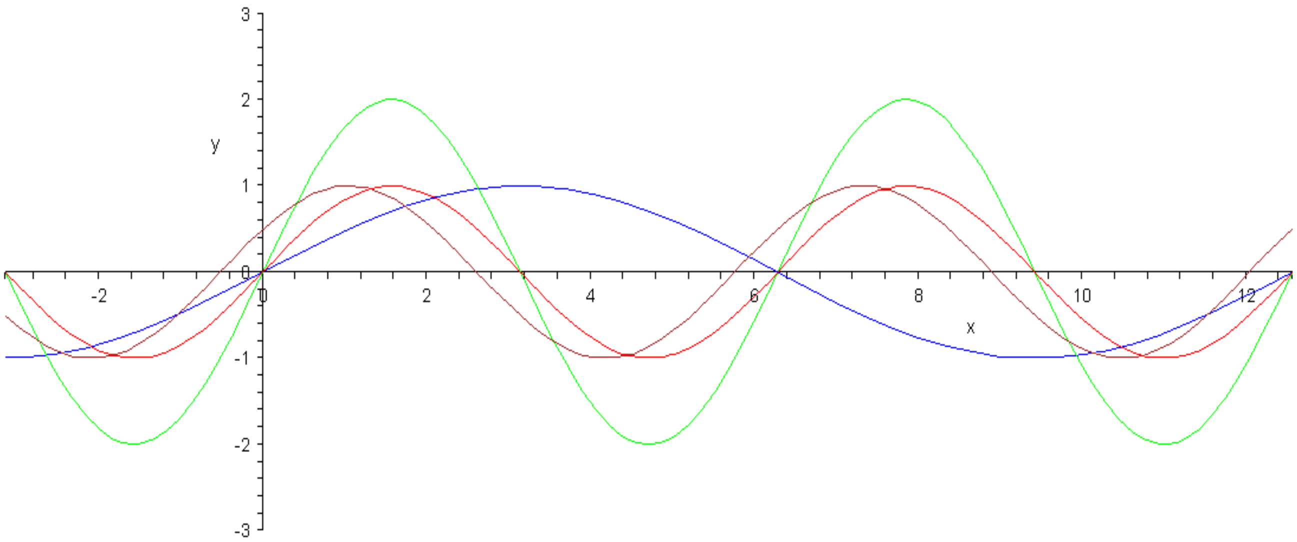
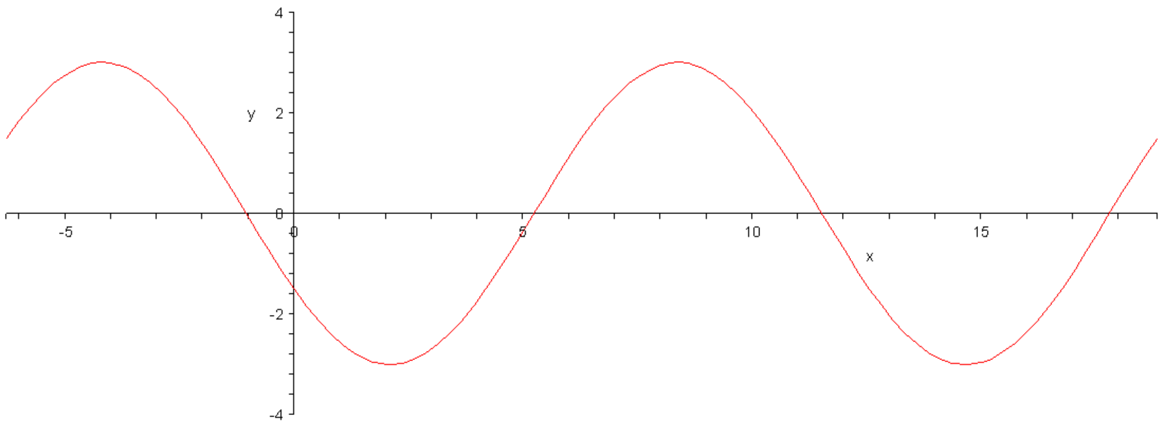
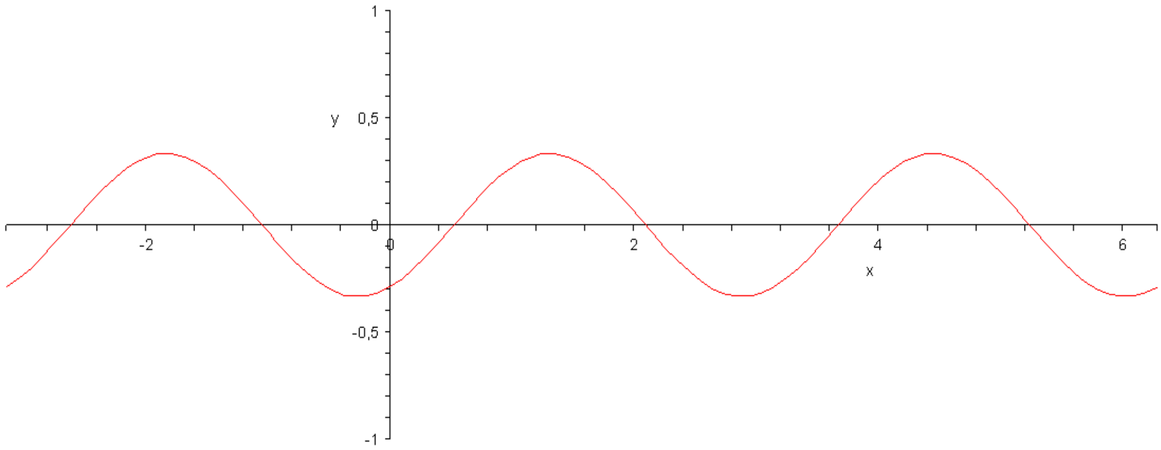
str. 41, př. 1
Nakreslete grafy funkcí y=sinx, y=sin(x/2), y=2sinx, y=sin(x+pi/6)



str. 41, př. 2
Nakreslete grafy funkcí y=3cos(x/2+2pi/3)

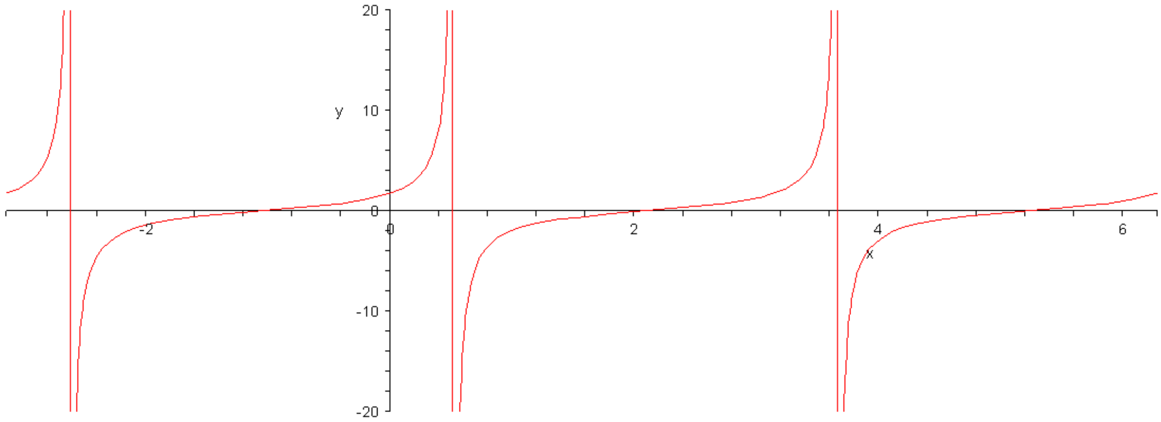


str. 41, př. 3
Nakreslete grafy funkcí y=1/3sin(2x-pi/3)



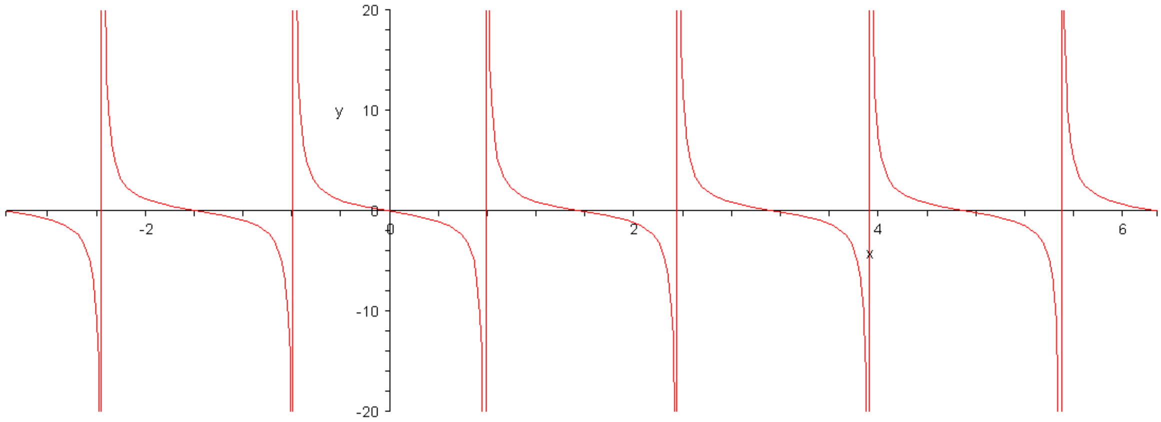
str. 41, př. 4

Nakreslete grafy funkcí y=tan(x+pi/3)



str. 41, př. 5

Nakreslete grafy funkcí y=cotg(2x+pi/2)



str. 41, př. 1

$$\lim_{x\to 0}\frac{1-\cos(x)}{x^2}=\lim_{x\to 0}\frac{\left(1-\cos(x)\right)'}{\left(x^2\right)'}=\lim_{x\to 0}\frac{\left(0+\sin(x)\right)'}{\left(2x\right)'}=\lim_{x\to 0}\frac{\cos(x)}{2}=\frac{1}{2}$$

$$\lim_{x\to \pi}\frac{\sin(5x)-\sin(3x)}{\sin(x)}=\lim_{x\to \pi}\frac{5\cos(5x)-3\cos(3x)}{\cos(x)}=\frac{-5+3}{-1}=2$$

$$\lim_{x\to 0}\frac{1-\cos^2(x)}{x}=\lim_{x\to 0}\frac{0+2\sin(x).\cos^2(x)}{1}=\frac{0}{1}=0$$

$$\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(x)}{\cos^2(x)}-\tan^2(x)\right)=\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(x)}{\cos^2(x)}-\frac{\sin^2(x)}{\cos^2(x)}\right)=\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(x)}{-2\cos(x).\sin(x)}-\frac{2\sin(x).\cos(x)}{2\cos(x).\sin(x)}\right)=$$

$$=\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(x)}{-2\cos(x).\sin(x)}+1\right)=\frac{1}{-2.1}+1=-\frac{1}{2}+\frac{2}{2}=\frac{1}{2}$$

$$\lim_{x\to \frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2}-\cos(x)}=\lim_{x\to \frac{\pi}{6}}\frac{1.\cos\left(x-\frac{\pi}{6}\right)}{0+\sin(x)}=\frac{1.1}{\frac{1}{2}}=2$$

$$\lim_{x\to\frac{\pi}{4}} \tan(2x).\tan\left(\frac{\pi}{4}-x\right) = \lim_{x\to\frac{\pi}{4}} \frac{\sin(2x).\tan\left(\frac{\pi}{4}-x\right)}{\cos(2x)} = \lim_{x\to\frac{\pi}{4}} \frac{2\cos(2x).\tan\left(\frac{\pi}{4}-x\right)-\sin(2x).\frac{1}{\cos\left(\frac{\pi}{4}-x\right)}}{-2\sin(2x)} =$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin(2x).\frac{\sin(\pi/4-x)}{\cos(\pi/4-x)}}{-2\sin(2x)} = \lim_{x\to\frac{\pi}{4}} \frac{\sin(2x)\frac{\sin(\pi/4-x)}{\cos(\pi/4-x)}}{\cos(2x)} = \lim_{x\to\frac{\pi}{4}} \frac{\sin(2x).\sin(\pi/4-x)}{\cos(2x).\cos(\pi/4-x)} =$$

$$\lim_{x\to\frac{\pi}{4}} \frac{2\cos x.\sin(\pi/4-x)-\sin(2x).\cos(\pi/4-x)}{-2\sin(2x)\cos(\pi/4-x)+\cos(2x)\sin(\pi/4-x)} = \frac{2.\frac{\sqrt{2}}{2}.0-1.1}{-2.1+1.0} = \frac{-1}{-2} = \frac{1}{2}$$

str. 41, př. 1

$$\int \sin^2(x)\,dx = \int \frac{1-\cos(2x)}{2}dx = \int \frac{1}{2}dx - \int \frac{\cos(2x)}{2}dx = \frac{1}{2}x - \frac{1}{2}\sin(2x) + C$$

str. 41, př. 2

$$\int \sin(x).\sin(5x)dx = \frac{1}{2}\int\left(\cos(-4x)-\cos(6x)\right)dx = \begin{vmatrix} -4x = a & dx = -da/4 \\ 6x = b & db = db/6 \end{vmatrix} = \frac{1}{2}\int\frac{\cos(a)}{-4}da - \frac{1}{2}\int\frac{\cos(b)}{6} =$$

$$= \frac{1}{8}\sin a - \frac{1}{12}\sin b = \frac{1}{8}\sin(-4x) - \frac{1}{12}\sin(6x) + C$$

str. 41, př. 3

$$\int \frac{1}{1+\cos x}dx = \int \frac{1}{1+\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}}dx = \begin{vmatrix} \tan(x/2) = t \\ (1/2)x = \arctan(x) \\ x = 2\arctan(x) \\ dx = \frac{2}{1+t^2}dt \end{vmatrix} = \int \frac{1}{1+\frac{1-t^2}{1+t^2}}.\frac{2}{1+t^2}dt = \int \frac{2}{1+t^2+\frac{1-t^2}{1+t^2}+\frac{t^2-t^4}{1+t^2}}dt =$$

$$\int \frac{2\left(1+t^2\right)}{1+t^2+t^2+t^4+1-t^2+t^2-t^4}dt = \int \frac{2\left(1+t^2\right)}{2+2t^2}dt = \int dt = t = tg\left(\frac{x}{2}\right) + C$$

str. 41, př. 4