

6.11.

$$1) f(z) = \frac{3z - \sqrt{z}}{z^2 + 2z - 3}; z_0 = 0$$

$$\sum_{k=0}^{\infty} \frac{z^k}{a^k} = \frac{z}{a-z}; |z| < |a|$$

$$D = 4 + 12 = 16$$

$$z_{1,2} = \frac{-2 \pm 4}{2} \begin{cases} \rightarrow -3 \\ \rightarrow 1 \end{cases}$$

$$A = \frac{3z - \sqrt{z}}{z-1} \Big|_{z=3} = \frac{-3 - \sqrt{3}}{-4} = \frac{-3 - \sqrt{3}}{-4} \cdot \frac{1}{2}$$

$$B = \frac{3z - \sqrt{z}}{z+3} \Big|_{z=1} = \frac{3 - \sqrt{1}}{4} = \frac{2}{4} = \frac{1}{2}$$

$$= \frac{3z - \sqrt{z}}{(z+3)(z-1)} = \frac{A}{z+3} + \frac{B}{z-1}$$

$$= \frac{\frac{1}{2}}{z+3} - \frac{\frac{1}{2}}{z-1} =$$

$$\frac{1}{2} \cdot \frac{1}{1 - (-\frac{z}{3})} + \frac{1}{2} \cdot \frac{1}{1-z} =$$

$$\frac{1}{6} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot (z)^k}{3^k} + \frac{1}{2} \sum_{k=0}^{\infty} z^k =$$

$$= \frac{1}{2} \cdot \left( \sum_{k=0}^{\infty} \frac{z \cdot (-1)^k \cdot z^k}{3^{k+1}} + \frac{z^{k+1}}{3^{k+1}} \cdot z^k \right)$$

$$= \frac{1}{2} \sum_{k=0}^{\infty} \frac{z \cdot (-1)^k + z^{k+1}}{3^{k+1}} \cdot z^k$$

$|z| < \min\{3, 3\}$

$|z| < 1$

$$2) f(z) = \frac{z}{z^2 + 3}; z_0 = 0$$

$$\frac{z}{a-z} = \sum_{k=0}^{\infty} \frac{z^k}{a^k} \quad (|z| < |a|)$$

$$\frac{z}{z^2 + 3} = \frac{z}{3} \cdot \frac{1}{1 - (-\frac{z^2}{3})} = \frac{z}{3} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{2k}}{3^k} = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{2k+1}}{3^{k+1}}$$

$$\left| -\frac{z^2}{3} \right| < 1 \Rightarrow |z| < \sqrt{3}$$

8.11  
 2)  $f(z) = \frac{\sqrt{z-1}}{z^2 - z - 12}$ ;  $z_0 = 1$  ( $z_0 - 1 = 0$ )

$$= \frac{\sqrt{z-1}}{(z+4)(z-3)} = \frac{A}{z+4} + \frac{B}{z-3}$$

$$= \frac{3}{z+4} + \frac{2}{z-3}$$

$$= \frac{3}{(z-1)+1+4} + \frac{2}{(z-1)+1-3} = \frac{3}{(z-1)+5} + \frac{2}{(z-1)-2}$$

$$= \frac{3}{\sqrt{5}} \cdot \frac{1}{\left(\frac{z-1}{\sqrt{5}}\right)+1} + \frac{2}{2} \cdot \frac{1}{\left(\frac{z-1}{2}\right)-1} = \frac{3}{\sqrt{5}} \cdot \frac{1}{1 - \left(\frac{1-z}{\sqrt{5}}\right)} - \frac{1}{1 - \left(\frac{z-1}{2}\right)} =$$

$$= \frac{3}{\sqrt{5}} \cdot \sum_{k=0}^{\infty} \frac{(1-z)^k}{5^k} - \sum_{k=0}^{\infty} \frac{(z-1)^k}{2^k} = \frac{3}{\sqrt{5}} \cdot \sum_{k=0}^{\infty} \frac{(1-z)^k \cdot 2^k}{\sqrt{5} \cdot 2^k} - \sum_{k=0}^{\infty} \frac{(z-1)^k \cdot \sqrt{5}^k}{2^k \cdot \sqrt{5}^k} =$$

$$= \frac{3}{\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot (z-1)^k \cdot 2^k}{10^k} - \sum_{k=0}^{\infty} \frac{(z-1)^k \cdot \sqrt{5}^k}{10^k} =$$

$$= \frac{3}{\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-2)^k - \sqrt{5}^{k+1}}{10^k} \cdot (z-1)^k = \sum_{k=0}^{\infty} \frac{3 \cdot (-2)^k - \sqrt{5}^{k+1}}{\sqrt{5} \cdot 10^k} \cdot (z-1)^k$$

Patru je m sprema. P. Zlbe naprasung!

$$\frac{a}{a-z} = \sum_{k=0}^{\infty} \frac{z^k}{a^k}$$

$$y = 1 + 4p = 49$$

$$z_{12} = \frac{1 \pm 7}{2} \rightarrow -4 \rightarrow +3$$

$$A = \frac{\sqrt{z-1}}{z-3} \Big|_{z=-4} = \frac{-21}{-7} = 3$$

$$B = \frac{\sqrt{z-1}}{z+4} \Big|_{z=3} = \frac{14}{7} = 2$$

6.11 d)  $f(z) = \frac{4z-3}{z+3}$ ;  $z_0 = 2-j$   $(4z-3):(z+3) = 4 - \frac{15}{z+3}$   
 $z_0 - 2 + j = 0$   $-(4z+12)$   
 $0 - 15$

$$4 - \frac{15}{z+3} = 4 - \frac{15}{(\sqrt{-j}) + (z_0 - 2 + j)} =$$

$$= 4 - \frac{15}{(\sqrt{-j})} \cdot \frac{1}{1 + \frac{(z-2+j)}{(\sqrt{-j})}} =$$

$z+3 = z_0 - 2 + j$   
 $\sqrt{-j} = 0$

$$= 4 - \frac{15}{\sqrt{-j}} \cdot \frac{1}{1 - \left(-\frac{z-2+j}{\sqrt{-j}}\right)} = 4 - \frac{15}{\sqrt{-j}} \cdot \sum_{k=0}^{\infty} \frac{(z-2+j)^k}{(\sqrt{-j})^k} =$$

$$= 4 - \frac{15}{\sqrt{-j}} \cdot \frac{\sqrt{-j}}{\sqrt{-j}} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k (z-2+j)^k}{(\sqrt{-j})^k} \cdot \frac{(\sqrt{-j})^k}{(\sqrt{-j})^k} =$$

$$= 4 - \frac{15 \cdot (\sqrt{-j})}{26} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k (\sqrt{-j})^k}{26^k} \cdot (z-2+j)^k =$$

$$= \underbrace{\frac{29 - 15j}{26}}_{k=0} + \sum_{k=1}^{\infty} \frac{15 \cdot (-1)^{k+1} \cdot (\sqrt{-j})^{k+1}}{26^{k+1}} \cdot (z-2+j)^k \quad \text{? (viel by die } z+1 \text{) alle } z \text{ lede}$$

$$\left| \frac{z-2+j}{\sqrt{-j}} \right| < 1$$

6.30

$$a) \sum_{k=1}^{\infty} \frac{7^k}{k \cdot \ln(k+1)} \quad ; \quad r = ?$$

$$= \lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = \frac{7^{k+1}}{(k+1) \cdot \ln(k+2)} \cdot \frac{k \cdot \ln(k+1)}{7^k} = \lim_{k \rightarrow \infty} \frac{7^{k+1}}{7^k} \cdot \frac{k \cdot \ln(k+1)}{(k+1) \cdot \ln(k+2)} =$$

$$= \lim_{k \rightarrow \infty} 7 \cdot \frac{k \cdot \ln(k+1)}{(k+1) \cdot \ln(k+2)} = 7 \cdot \lim_{k \rightarrow \infty} \frac{k \cdot \ln(k+1)}{(k+1) \cdot \ln(k+2)}$$

$$\Rightarrow \lim_{k \rightarrow \infty} \frac{k \cdot \ln(k+1)}{(k+1) \cdot \ln(k+2)} \stackrel{\text{IH}}{=} \lim_{k \rightarrow \infty} \frac{\ln(k+1) + \frac{1}{k+1}}{\ln(k+2) + \frac{1}{k+2}} =$$

$$= \lim_{k \rightarrow \infty} \frac{\ln\left(1 + \frac{1}{k}\right) + \frac{1}{1 + \frac{1}{k}}}{\ln\left(1 + \frac{2}{k}\right) + \frac{1}{1 + \frac{2}{k}}} = \frac{\ln(1+0) + \frac{1}{1+0}}{\ln(1+0) + \frac{1}{1+0}} =$$

$$= \frac{0+1}{0+1} = 1 \quad \Rightarrow \quad r = 1$$

$$b) \sum_{k=0}^{\infty} \frac{7^k}{\sqrt{2^{k+1}}} \quad ; \quad r = ?$$

$$r = \lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \rightarrow \infty} \frac{7^{k+1}}{\sqrt{2^{k+2}}} \cdot \frac{\sqrt{2^{k+1}}}{7^k} = \lim_{k \rightarrow \infty} \frac{7^{k+1}}{7^k} \cdot \frac{\sqrt{2^{k+2}}}{\sqrt{2^{k+1}}} =$$

$$= \lim_{k \rightarrow \infty} 7 \cdot 2^{\frac{k+2-k-1}{2}} = 7 \cdot \lim_{k \rightarrow \infty} 2^{\frac{1}{2}} = \boxed{2 = r}$$

6.30

$$c) \sum_{k=0}^{\infty} (2^k + (-1)^{k+1}) 7^{-k}; \quad r = ?$$

$$\sum_{k=0}^{\infty} (2^k) 7^{-k} - \sum_{k=0}^{\infty} (-2)^k \rightarrow \text{geometrica' rada}$$

$$q_1 = 2/7; \quad q_2 = -2$$

$$|q_1| < \frac{1}{2}, \quad |q_2| < 1 \Rightarrow r = \min\{r_1; r_2\} =$$

$$= \min\left\{\frac{1}{2}; 1\right\} \Rightarrow r = \frac{1}{2}$$

$$d) \sum_{k=0}^{\infty} \frac{7^{2k+1}}{\sqrt{k+1}}; \quad r = ?$$

$$A = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \lim_{k \rightarrow \infty} \frac{7^{2k+3}}{\sqrt{k+2}} \cdot \frac{\sqrt{k+1}}{7^{2k+1}} = \lim_{k \rightarrow \infty} \frac{7^{2k+3}}{7^{2k+1}} \cdot \frac{\sqrt{k+1}}{\sqrt{k+2}} =$$

$$= \lim_{k \rightarrow \infty} 7^{2k+3-2k-1} \cdot \frac{\sqrt{1 + \frac{1}{k}}}{\sqrt{1 + \frac{2}{k}}} = |7|^2 \cdot \lim_{k \rightarrow \infty} \frac{\sqrt{1+0}}{\sqrt{1+0}} = 49$$

$$\boxed{r = 49}$$

6.31

$$a) \sum_{k=0}^{\infty} \frac{(z-1)^{2k}}{(k+1)^2}; \quad r: \text{obor}$$

$$A = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \lim_{k \rightarrow \infty} \frac{(z-1)^{2k+2}}{(k+2)^2} \cdot \frac{(k+1)^2}{(z-1)^{2k+1}} = \lim_{k \rightarrow \infty} (z-1)^{2k+2-2k-1} \cdot \frac{(k+1)^2}{(k+2)^2} =$$

$$= (z-1) \cdot \lim_{k \rightarrow \infty} \frac{(k+2)^2}{(k+1)^2} = (z-1) \cdot \lim_{k \rightarrow \infty} \frac{k^2 + 4k + 4}{k^2 + 2k + 1} \stackrel{(\frac{0}{0})^2}{=} 1$$

$$= (z-1) \cdot \lim_{k \rightarrow \infty} \frac{2}{2} = 1 \Rightarrow r = 1$$

$$\text{obor } |z-1| \leq 1$$

$$b) \sum_{k=2}^{\infty} \frac{(z+j)^k}{3^k}; \quad r: \text{obor?}$$

$$A = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \lim_{k \rightarrow \infty} \frac{(z+j)^{k+1}}{3^{k+1}} \cdot \frac{3^k}{(z+j)^k} = \lim_{k \rightarrow \infty} \frac{(z+j)^{k+1}}{(z+j)^k} \cdot \frac{3^k}{3^{k+1}} =$$

$$= \lim_{k \rightarrow \infty} (z+j)^{k+1-k} \cdot \frac{3^k}{3^{k+1}} = (z+j) \cdot \lim_{k \rightarrow \infty} \frac{1}{3} = \frac{z+j}{3} \Rightarrow$$

$$\frac{|z+j|}{3} \leq 1$$

$$\boxed{|z+j| \leq 3 \Rightarrow r = 3 \text{ P.}}$$

6.21.

$$a) \sum_{k=1}^{\infty} \left( \frac{r-1}{2j+1} \right)^{3k} \rightarrow r; 0 \text{ boz?}$$

$$A = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \frac{\left( \frac{r-1}{2j+1} \right)^{3k+1}}{\left( \frac{r-1}{2j+1} \right)^{3k}} = \left( \frac{r-1}{2j+1} \right) \cdot \lim_{k \rightarrow \infty} 1$$

$$\left| \frac{r-1}{2j+1} \right| \leq 1 \Rightarrow |r-1| \leq |2j+1| = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\text{oboz} \rightarrow \boxed{|r-1| \leq \sqrt{5}} \quad \boxed{r = \sqrt{5}}$$

6.22. Unči T. Raah

$$a) f(x) = \frac{1 - \cos x}{x^2}; \quad f(0) = \frac{1}{2}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!} \quad x \in \mathbb{R}$$

$$\begin{aligned} f(x) &= \frac{1}{x^2} \left( 1 - \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!} \right) = \frac{1}{x^2} \cdot \left( 1 + (-1) \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!} \right) = \\ &= \frac{1}{x^2} \left( 1 + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot x^{2k}}{(2k)!} \right) = x^{-2} \cdot \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^{2k}}{(2k)!} = \end{aligned}$$

$k=0$  je suma = -1

$1 + (-1) = 0 \Rightarrow$  razenimo  $k=1$  o

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^{2(L-1)}}{(2k)!}$$

6.32 b)  $f(x) = \frac{e^{2x} - 1}{x}$  ;  $f(0) = 2$   $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad x \in \mathbb{C}$

$$f(x) = \frac{1}{x} \cdot \left( \sum_{k=0}^{\infty} \frac{2^k x^k}{k!} - 1 \right) = \frac{1}{x} \sum_{k=1}^{\infty} \frac{2^k x^k}{k!} =$$

$$= \sum_{k=1}^{\infty} \frac{2^k x^{k-1}}{k!}$$

c)  $f(x) = \sin^2 x = \frac{1}{2} (1 - \cos 2x)$   $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$

$$f(x) = \frac{1}{2} \left( 1 - \sum_{k=0}^{\infty} \frac{(-1)^k (2x)^{2k}}{(2k)!} \right) = \frac{1}{2} \left( \sum_{k=1}^{\infty} \frac{(-1)^{k+1} (2x)^{2k}}{(2k)!} \right) =$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot 2^{2k-1} \cdot x^{2k}}{(2k)!}$$

x)  $\sin x \cdot \cos 3x = ? \quad x \in \mathbb{R}$

$x = \alpha$   
 $3x = \beta$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \quad x \in \mathbb{R}$$

$$\cos 3x = \sum_{k=0}^{\infty} \frac{(-1)^k (3x)^{2k}}{(2k)!}$$

$$\sin x \cdot \cos 3x = \frac{1}{2} (\sin 4x - \sin 2x) = \frac{1}{2} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (4x)^{2k+1}}{(2k+1)!} - \sum_{k=0}^{\infty} \frac{(-1)^k (2x)^{2k+1}}{(2k+1)!} \right) =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 4 \cdot (2 \cdot 4 - 1) x^{2k+1}}{(2k+1)!}$$

$\sin(x+\beta) = \sin x \cos \beta + \sin \beta \cos x$   
 $\sin(x-\beta) = \sin x \cos \beta - \sin \beta \cos x$   
 $\sin(x+\beta) + \sin(x-\beta) = 2 \sin x \cos \beta$   
 $\frac{\sin(x+3x) + \sin(x-3x)}{2} = \sin x \cos 3x$   
 $\frac{1}{2} (\sin 4x + \sin(-2x)) = \sin x \cos 3x$   
 $\frac{1}{2} (\sin 4x - \sin 2x) = \sin x \cos 3x$



6.32

e)  $\sinh x, x \in \mathbb{R}$      $\sinh x = \frac{1}{2}(e^x - e^{-x})$  ;  $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$\begin{aligned} \sinh x &= \frac{1}{2} \left( \sum_{k=0}^{\infty} \frac{x^k}{k!} - \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!} \right) = \\ &= \frac{1}{2} \left( \sum_{k=0}^{\infty} \frac{x^k}{k!} + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot x^k}{k!} \right) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k)!} = \\ &= \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k)!} \end{aligned}$$

f)  $\cosh x = \frac{1}{2}(e^x + e^{-x})$  ;  $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$\begin{aligned} \cosh x &= \frac{1}{2}(e^x + e^{-x}) = \frac{1}{2} \left( \sum_{k=0}^{\infty} \frac{x^k}{k!} + \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!} \right) = \\ &= \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} \end{aligned}$$

6.33.  $f(x) = \int_0^x \frac{\sinh t}{t} dt =$

~~$\sinh x = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{(2k+1)!}$~~

$$\begin{aligned} &= \int_0^x \frac{1}{t} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^{2k+1}}{(2k+1)!} dt = \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^{2k}}{(2k+1)!} dt = \\ &= \sum_{k=0}^{\infty} \int_0^x \frac{(-1)^k \cdot t^{2k}}{(2k+1)!} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left[ \frac{t^{2k+1}}{2k+1} \right]_0^x = \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{(2k+1)! \cdot (2k+1)} ; x \in \mathbb{R} \end{aligned}$$

6.27.

$$\begin{aligned}
 f(x) &= \int_0^x \frac{1-e^{-t}}{t} dt = \int_0^x \frac{1}{t} (1-e^{-t}) dt = \\
 &= \int_0^x \frac{1}{t} \left( 1 - \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^k}{k!} \right) dt = \int_0^x \frac{1}{t} \left( 1 + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot t^{k+1}}{(k+1)!} \right) dt = \\
 &= \int_0^x \frac{1}{t} \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot t^{k+1}}{(k+1)!} dt = \sum_{k=1}^{\infty} \int_0^x \frac{(-1)^{k+1} \cdot t^{k+1}}{(k+1)!} dt = \\
 &= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k+1)!} \left[ \frac{t^{k+2}}{(k+2)} \right]_0^x = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^{k+2}}{(k+2) \cdot (k+1)!}
 \end{aligned}$$

$$\begin{aligned}
 c) f(x) &= \int_0^x (1+t^4)^{-\frac{1}{2}} dt = (1+x)^{-\frac{1}{2}} = \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} x^k \\
 &= \int_0^x \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} t^{4k} dt = \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} \int_0^x t^{4k} dt = \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} \left[ \frac{t^{4k+1}}{4k+1} \right]_0^x = \\
 &= \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} \frac{x^{4k+1}}{4k+1} = x + \sum_{k=1}^{\infty} \frac{(-1)^k \cdot (2k-1)!!}{(2k)!! \cdot (k+1)} x^{4k+1}
 \end{aligned}$$

F. RAD

$$7.14. a) f(t) = \cos t ; t \in (0; \frac{\pi}{2}) \Rightarrow T = \frac{2\pi}{\omega} ; \omega = \frac{2\pi}{T} = 4$$

$$a_0 = \frac{2}{T} \int_a^{a+T} f(x) dx$$

$$a_0 = \frac{2}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos t dt = \frac{4}{\pi} \cdot [\sin t]_0^{\frac{\pi}{2}} = \frac{4}{\pi} \cdot 1 = \frac{4}{\pi}$$

$$a_L = \frac{2}{T} \int_a^{a+T} f(t) \cdot \cos L \cdot \omega t dt =$$

$$\Rightarrow \frac{2}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos t \cdot \cos 4t dt \Rightarrow$$

$$\cos(x+\beta) = \cos x \cdot \cos \beta - \sin x \cdot \sin \beta \quad x = 4t$$

$$\cos(x-\beta) = -\cos x \cdot \cos \beta + \sin x \cdot \sin \beta \quad \beta = t$$

$$\cos(x+\beta) + \cos(x-\beta) = 2 \cdot \cos x \cdot \cos \beta \Rightarrow$$

$$\cos t \cdot \cos 4t = \frac{1}{2} \cdot [\cos(4t+t) + \cos(4t-t)]$$

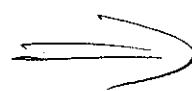
$$\Rightarrow \frac{1}{2} \cdot \frac{4}{\pi} \int_0^{\frac{\pi}{2}} (\cos[(4t+1)t] + \cos[(4t-1)t]) dt =$$

$$= \frac{2}{\pi} \left[ \frac{\sin[(4t+1)t]}{4t+1} + \frac{\sin[(4t-1)t]}{4t-1} \right]_0^{\frac{\pi}{2}} =$$

$$= \frac{2}{\pi} \left[ \frac{\sin(4t+1) \cdot \frac{\pi}{2}}{4t+1} - \frac{\sin(4t-1) \cdot \frac{\pi}{2}}{4t-1} \right] = \sin\left(\frac{2t}{2} + \frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1$$

Periody 0

$$= \frac{2}{\pi} \left[ \frac{1}{4t+1} - \frac{1}{4t-1} \right] = \frac{2}{\pi} \left( \frac{4t-1 - 4t-1}{16t^2-1} \right) = \frac{-2}{\pi \cdot (16t^2-1)}$$



$$A_{\text{eff}} = \frac{2}{T} \int_0^{T/2} f(t) \cdot \sin 4kt \, dt =$$

$$= \frac{2}{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} 2 \cdot \cos t \cdot \sin 4kt \, dt =$$

$$\begin{aligned} \sin(x+\beta) &= \sin x \cdot \cos \beta + \cos x \cdot \sin \beta \\ \sin(x-\beta) &= \sin x \cdot \cos \beta - \cos x \cdot \sin \beta \\ \sin(x+\beta) + \sin(x-\beta) &= 2 \cdot \sin x \cdot \cos \beta \\ \alpha &= 4kt; \beta = t \end{aligned}$$

$$= \frac{1}{2} \frac{4}{\pi} \int_0^{\frac{\pi}{4}} \left[ \sin(4kt+t) + \sin(4kt-t) \right] dt =$$

$$= \frac{2}{\pi} \left[ -\frac{\cos[(4k+1)t]}{4k+1} - \frac{\cos[(4k-1)t]}{4k-1} \right]_0^{\frac{\pi}{4}} =$$

$$= -\frac{2}{\pi} \left[ \frac{\cos\left(\frac{4k\pi}{2} + \frac{\pi}{2}\right)}{4k+1} + \frac{\cos\left(\frac{4k\pi}{2} - \frac{\pi}{2}\right)}{4k-1} \right] =$$

$$= \frac{2}{\pi} \left[ \frac{-\cos 0}{4k+1} - \frac{\cos 0}{4k-1} \right] = 0 + \frac{2}{\pi} \left( \frac{1}{4k+1} + \frac{1}{4k-1} \right) =$$

$$= \frac{2}{\pi} \left( \frac{4k-1 + 4k+1}{16k^2-1} \right) = \frac{2 \cdot 8k}{\pi \cdot (16k^2-1)} = \frac{16k}{\pi \cdot (16k^2-1)}$$

F.R.

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cdot \cos k\omega t + b_k \cdot \sin k\omega t) =$$

$$= \frac{2}{\pi} + \sum_{k=1}^{\infty} \left( -\frac{4}{\pi \cdot (16k^2-1)} \cdot \cos 4kt + \frac{16k}{\pi \cdot (16k^2-1)} \cdot \sin 4kt \right) =$$

$$= \frac{2}{\pi} + \sum_{k=1}^{\infty} \frac{4}{\pi \cdot (16k^2-1)} \cdot (4k \sin 4kt - \cos 4kt)$$

7.14 b)  $f(t) = t^2 \quad t \in (-1; 1) \quad T = 2 \quad ; \quad \omega = \frac{2\pi}{T} = \pi$

$$a_0 = \frac{2}{T} \int_{-1}^{1} t^2 dt = 1 \cdot \int_{-1}^1 \frac{1}{3} t^3 = \left[ \frac{1}{3} t^3 \right]_{-1}^1 = \frac{1}{3} - \frac{(-1)}{3} = \left[ \frac{2}{3} \right]_{a_0}$$

$$a_k = \frac{2}{T} \int_{-1}^{1} t^2 \cdot \cos k\omega t dt = \int_{-1}^1 t^2 \cdot \cos k\pi t dt = \left. \begin{array}{l} u = t^2 \\ u' = 2t \\ v = \cos k\pi t \\ v' = -\sin k\pi t \end{array} \right|$$

$$= \left. \frac{t^2 \cdot \sin k\pi t}{k\pi} \right|_{-1}^1 - \left. \frac{2}{k\pi} \int_{-1}^1 t \cdot \sin k\pi t dt \right|_{u=+} \left. \begin{array}{l} u = t \\ u' = 1 \\ v = \cos k\pi t \\ v' = -\sin k\pi t \end{array} \right|$$

$$= \left. \frac{t^2 \cdot \sin k\pi t}{k\pi} \right|_{-1}^1 + \frac{2}{k\pi} \left[ \left. \frac{t \cdot \cos k\pi t}{k\pi} \right|_{-1}^1 + \left. \frac{1}{k\pi} \int_{-1}^1 \cos k\pi t dt \right|_{-1}^1 \right]$$

$$= \left[ \frac{t^2 \cdot \sin k\pi t}{k\pi} + \frac{2}{k\pi} \left[ \frac{t \cdot \cos k\pi t}{k\pi} + \frac{1}{k\pi} \cdot \frac{\sin k\pi t}{k\pi} \right] \right]_{-1}^1 =$$

$$= \left( \frac{t^2 \cdot \sin k\pi t}{k\pi} + 2t \cdot \frac{\cos k\pi t}{k^2 \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin k\pi t \right)_{-1}^1 =$$

$$= \frac{1 \cdot \sin k\pi}{k\pi} + \frac{2 \cdot \cos k\pi}{k^2 \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin k\pi$$

$$\left[ \frac{1 \cdot \sin(-k\pi)}{k\pi} - \frac{2 \cdot \cos(-k\pi)}{k^2 \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin(-k\pi) \right] =$$

$$= 0 + \frac{2 \cdot (-1)}{k^2 \pi^2} + 0 - \left( \frac{0}{k\pi} - \frac{2 \cdot (-1)}{k^2 \pi^2} + 0 \right) =$$

$$= \frac{-4}{k^2 \pi^2} \Rightarrow \frac{(-1)^k \cdot 4}{k^2 \pi^2}$$

$\Rightarrow$

$$A_2 = \frac{2}{T} \int_0^{x_4 T} t^2 \sin \omega t dt =$$

$$= \int_{-1}^1 t^2 \sin \omega t dt \Rightarrow \text{mit Integration!}$$

$$\left[ t^2 \cdot \frac{(-\cos \omega t)}{\omega} + 2t \cdot \frac{(\sin \omega t)}{\omega^2} + \frac{2}{\omega^3} \cdot (-\cos \omega t) \right]_{-1}^1 =$$

$$= \left( \frac{t^2 \cdot \cos \omega t}{\omega} + 0 + \frac{2}{\omega^3} \cdot (-\cos \omega t) \right) -$$

$$- \left( \frac{t^2}{\omega} \cdot 1 + 0 + \frac{2}{\omega^3} \cdot 1 \right) = 0 = b_2$$

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos \omega t + b_k \cdot \sin \omega t =$$

$$= \frac{1}{2} + \sum_{k=1}^{\infty} \frac{(-1)^k \cdot 4}{k^2 \cdot \pi^2} \cdot \cos \omega t$$

7.3P

$$a) f(t) = 3 \sin^2 t + 2 \cos^2 t = 3 \cdot \frac{1}{2} (1 - \cos 2t) + \frac{2}{2} (1 + \cos 2t) = \frac{3}{2} - \frac{3}{2} \cos 2t + 1 + \cos 2t = \frac{5}{2} - \frac{1}{2} \cos 2t$$

$$b) f(t) = \sqrt{t} \cos 3t = \sqrt{t} \cos \cdot \frac{1}{2} (1 + \cos 2t) = \frac{\sqrt{t}}{2} \cos t + \frac{\sqrt{t}}{2} \cos t \cdot \cos 2t = \frac{\sqrt{t}}{2} \cos t + \frac{\sqrt{t}}{2} \left( \frac{\cos 3t + \cos(-t)}{2} \right) = \frac{\sqrt{t}}{2} \cos t + \frac{\sqrt{t}}{4} (\cos 3t + \cos t) = \frac{1\sqrt{t}}{4} \cos t + \frac{\sqrt{t}}{4} \cos 3t$$

$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cdot \cos \beta$

$$c) f(t) = 7 \cdot \sin^4 t$$

$$\sin^4 t = \frac{(1 - \cos^2 t)}{2} \cdot \frac{(1 - \cos^2 t)}{2} = \frac{1}{4} (1 - 2 \cos^2 t + \cos^2 2t) = \frac{1}{4} \left( 1 - 2 \cos^2 t + \frac{1}{2} (1 + \cos 4t) \right) = \frac{1}{4} \left( 1 - 2 \cos^2 t + \frac{1}{2} + \frac{1}{2} \cos 4t \right) = \frac{1}{4} - \frac{1}{2} \cos^2 t + \frac{1}{8} + \frac{1}{8} \cos 4t = \frac{3}{8} - \frac{1}{2} \cos^2 t + \frac{1}{8} \cos 4t$$

$$\Rightarrow 7 \cdot \sin^4 t = \frac{7 \cdot 3}{8} - 7 \cdot \frac{1}{2} \cos^2 t + 7 \cdot \frac{1}{8} \cos 4t = \frac{21}{8} - \frac{7}{2} \cos^2 t + \frac{7}{8} \cos 4t$$

$$d) \sqrt{t} \sin t \cdot \cos 3t = \sqrt{t} \left( \sin 4t + \sin(-2t) \right) = \frac{\sqrt{t}}{2} \sin 4t - \frac{\sqrt{t}}{2} \sin 2t$$

$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cdot \cos \beta$

$$e) f(t) = P \sin 3t \cdot \sin 2t = \frac{P}{2} (\cos(t) - \cos(5t)) = \frac{P}{2} (4 \cos t - 4 \cos 5t)$$

$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$   
 $\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$   
 $\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2 \sin \alpha \cdot \sin \beta$

$$f) 15 \cos 4t \cdot \cos t = \frac{15}{2} (\cos 5t + \cos 3t)$$

$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cdot \cos \beta$

7.14  
 c)  $f(t) = t + \varepsilon(-\tau; \tau)$      $T = 2\tau$      $\xi = \frac{2\tau}{T} = 1$

$$a_0 = \int_{-\tau}^{\tau} f(t) dt = \int_{-\tau}^{\tau} t dt = \left[ \frac{1}{2} t^2 \right]_{-\tau}^{\tau} = \frac{1}{2} \tau^2 - \frac{1}{2} (-\tau)^2 = 0$$

$$a_k = \frac{2}{2\tau} \int_{-\tau}^{\tau} t \cdot \cos \xi t dt = \left[ \begin{array}{l} u = t \quad v = \cos \xi t \\ u' = 1 \quad v' = -\frac{\sin \xi t}{\xi} \end{array} \right] =$$

$$= \frac{1}{\tau} \left[ t \cdot \frac{\sin \xi t}{\xi} - \int_{-\tau}^{\tau} \sin \xi t dt \right] =$$

$$= \frac{1}{\tau} \left[ t \cdot \frac{\sin \xi t}{\xi} + \frac{1}{\xi^2} \cos \xi t \right]_{-\tau}^{\tau} = \frac{1}{\tau} \left[ 0 + \frac{(-1)}{\xi^2} + 0 - \frac{(-1)}{\xi^2} \right] = 0$$

$$b_k = \frac{2}{2\tau} \int_{-\tau}^{\tau} t \cdot \sin \xi t dt = \frac{1}{\tau} \left[ t \cdot \frac{(-\cos \xi t)}{\xi} + \frac{1}{\xi^2} \sin \xi t \right]_{-\tau}^{\tau} =$$

$$= \frac{1}{\tau} \left( \frac{\tau(-\cos \xi \tau)}{\xi} - \frac{(-\tau)}{\xi} (-\cos(-\tau \xi)) \right) =$$

$$= \frac{(-\cos \xi \tau)}{\xi} + \frac{\cos(-\tau \xi)}{\xi} = -\frac{2 \cos(\xi \tau)}{\xi}$$

$$\frac{3}{2} (1 -$$

$$3 \sin^2 t + 2(1 - \sin^2 t) = 2 + \sin^2 t = 2 + \frac{1}{2} (1 + \cos 2t)$$

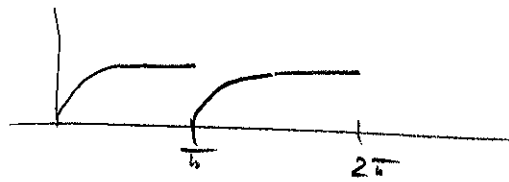
$$\int \cos 2t \left( \frac{1}{2} + \cos 2t \right) dt = \frac{t}{2} - \frac{\cos 2t}{2}$$

=



7.29

$$a) f(t) = \begin{cases} \sin t + \varepsilon < 0; \frac{\pi}{2} > \\ 1 & + \varepsilon < \frac{\pi}{2}; \pi > \end{cases}$$



$$T = \pi \quad ; \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cdot \cos k\omega t \, dt$$

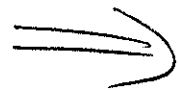
$$b_k = \frac{2}{T} \int_0^T f(t) \cdot \sin k\omega t \, dt$$

$$FR: \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega t + b_k \sin k\omega t)$$

$$\begin{aligned} a_0 &= \frac{2}{T} \left( \int_0^{\frac{\pi}{2}} \sin t \, dt + \int_{\frac{\pi}{2}}^{\pi} 1 \, dt \right) = \frac{2}{\pi} \left( \left[ -\cos t \right]_0^{\frac{\pi}{2}} + \left[ t \right]_{\frac{\pi}{2}}^{\pi} \right) \\ &= \frac{2}{\pi} \left( -\cos \frac{\pi}{2} + \cos 0 + \pi - \frac{\pi}{2} \right) = \frac{2}{\pi} \left( 0 + 1 + \pi - \frac{\pi}{2} \right) = \frac{2}{\pi} \left( 1 + \frac{\pi}{2} \right) \\ &= \frac{2}{\pi} + \frac{2\pi}{2\pi} = \frac{2+\pi}{\pi} \end{aligned}$$

$$\sin(x+\beta) + \sin(x-\beta) = 2 \sin x \cdot \cos \beta$$

$$\begin{aligned} a_k &= \frac{2}{\pi} \cdot \int_0^{\frac{\pi}{2}} \sin t \cdot \cos 2kt \, dt + \int_{\frac{\pi}{2}}^{\pi} \cos 2kt \, dt \\ &= \frac{2}{\pi} \cdot \left( \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin(2k+1)t + \sin(1-2k)t \, dt + \int_{\frac{\pi}{2}}^{\pi} \cos 2kt \, dt \right) \\ &= \frac{2}{\pi} \cdot \left( \frac{1}{2} \left[ \frac{-\cos(2k+1)t}{2k+1} - \frac{\cos(1-2k)t}{1-2k} \right]_0^{\frac{\pi}{2}} + \left[ \frac{\sin 2kt}{2k} \right]_{\frac{\pi}{2}}^{\pi} \right) \\ &= \frac{2}{\pi} \cdot \left[ \frac{1}{2} \left[ \frac{-0}{2k+1} + \frac{-1}{2k+1} \right] + \left[ \frac{0}{1-2k} + \frac{-1}{1-2k} \right] + \left( \frac{-0 \neq 0}{2k} \right) \right] \\ &= \frac{2}{\pi} \cdot \left[ \frac{1}{2} \left( \frac{-1}{2k+1} + \frac{-1}{1-2k} \right) \right] = \frac{1}{\pi} \cdot \frac{-1+2k+2k+1}{1-4k^2} = \frac{-2}{\pi(1-4k^2)} \end{aligned}$$



7.38 702.

$$\begin{aligned} -\cos(x+\pi) &= \cos(x) \cos(\pi) + \sin(x) \sin(\pi) \\ (\cos(x-\pi) - \cos(x+\pi)) &= 2\sin(x) \sin(\pi) \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{2}{\pi} \left[ \int_0^{\frac{\pi}{2}} \sin 2kx \, dx + \int_{\frac{\pi}{2}}^{\pi} \sin 2kx \, dx \right] \\ &= \frac{2}{\pi} \left[ \int_0^{\frac{\pi}{2}} (\sin(1-2k)x + \sin(1+2k)x) \, dx + \int_{\frac{\pi}{2}}^{\pi} \sin 2kx \, dx \right] \\ &= \frac{2}{\pi} \left[ \left[ \frac{-\cos(1-2k)x}{1-2k} + \frac{+\cos(1+2k)x}{1+2k} \right]_0^{\frac{\pi}{2}} + \left[ \frac{-\cos 2kx}{2k} \right]_{\frac{\pi}{2}}^{\pi} \right] \\ &= \frac{2}{\pi} \left[ \left( \frac{+1}{1-2k} - \frac{1}{1+2k} \right) + 2 \left( \frac{-1}{2k} - \frac{1}{2k} \right) \right] \\ &= \frac{2}{\pi} \left[ \frac{1+2k-1+2k}{1-4k^2} - \frac{2 \cdot 2}{2k} \right] = \frac{2}{\pi} \left( \frac{4k}{1-4k^2} - \frac{2}{k} \right) \\ &= \frac{4}{\pi} \left( \frac{k}{1-4k^2} - \frac{1}{2k} \right) \end{aligned}$$

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Radik

$$f(x) = \frac{2+\pi}{2\pi} + \sum_{k=1}^{\infty} \frac{-2}{\pi \cdot (1-4k^2)} \cdot \cos 2kx + \left( \frac{4k(-1)^{k+1}}{\pi(1-4k^2)} + \frac{1-(-1)^k}{\pi k} \right) \sin 2kx$$

438

7.09  $f(t) = \begin{cases} \sin t & + \varepsilon \left(0; \frac{\pi}{2}\right) \\ 0 & + \varepsilon \left(\frac{\pi}{2}; \pi\right) \end{cases} \Rightarrow T = \pi, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

$$a_0 = \frac{2}{T} \int_0^{\frac{\pi}{2}} \sin t \, dt + \int_{\frac{\pi}{2}}^{\pi} 0 \, dt = \frac{2}{\pi} \left[ -\cos t \right]_0^{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$a_k = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin t \cdot \sin 2kt \, dt + \int_{\frac{\pi}{2}}^{\pi} 0 \cdot \sin 2kt \, dt = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin t \cdot \sin 2kt \, dt$$

$-\cos(\alpha+\beta) = -\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

$$= \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \left[ \frac{1}{2} \left( \cos(1-2k)t + \cos(1+2k)t \right) \right] dt = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos(\alpha-\beta) - \cos(\alpha+\beta) = 2 \sin\alpha \cdot \sin\beta$$

$$= \frac{1}{\pi} \left[ \frac{\sin(1-2k)t}{1-2k} - \frac{\sin(1+2k)t}{1+2k} \right]_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{\pi} \left( \frac{1}{1-2k} - \frac{1}{1+2k} \right) = \frac{1}{\pi} \left( \frac{1+2k - 1+2k}{1-4k^2} \right) = \frac{4k}{\pi(1-4k^2)}$$

$$a_k = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin t \cdot \cos 2kt \, dt =$$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \sin(1+2k)t + \sin(1-2k)t \, dt =$$

$$= \frac{1}{\pi} \left[ \frac{-\cos(1+2k)t}{1+2k} + \frac{\cos(1+2k)t}{1-2k} \right]_0^{\frac{\pi}{2}} = \frac{1}{\pi} \left( \frac{-1}{1+2k} + \frac{-1}{1-2k} \right) =$$

$$= \frac{-2}{\pi(1-4k^2)}$$

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos 2kt + b_k \cdot \sin 2kt =$$

$$= \frac{1}{\pi} + \sum_{k=1}^{\infty} \left( \frac{4k}{\pi(1-4k^2)} \sin 2kt + \frac{2}{\pi(1-4k^2)} \cos 2kt \right)$$

$$= \frac{1}{\pi} + \sum_{k=1}^{\infty} \left( \frac{(-1)^k - 1}{\pi(1-4k^2)} + \frac{4k}{\pi(1-4k^2)} \sin 2kt \right) + \varepsilon_R$$

7.19

c)  $f(t) = \begin{cases} \cos t & t \in (0; \frac{T}{2}) \\ 0 & t \in (\frac{T}{2}; T) \end{cases} \Rightarrow T = T \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

$a_0 = \frac{2}{T} \cdot \int_0^{\frac{T}{2}} \cos t dt = \frac{2}{T} \cdot [\sin t]_0^{\frac{T}{2}} = \frac{2}{T} \cdot 1 = \frac{2}{\pi}$

$a_{2k} = \frac{2}{T} \cdot \int_0^{\frac{T}{2}} \cos t \cdot \cos 2kt dt = \frac{2}{2\pi} \int_0^{\frac{\pi}{2}} \cos t \cdot \cos 2kt dt$   
 $\cos(x+\beta) + \cos(x-\beta) = 2\cos x \cdot \cos \beta$

$= \frac{2}{2\pi} \int_0^{\frac{\pi}{2}} (\cos(1+2k)t + \cos(1-2k)t) dt =$

$= \frac{2}{2\pi} \left[ \frac{\sin(1+2k)t}{1+2k} + \frac{\sin(1-2k)t}{1-2k} \right]_0^{\frac{\pi}{2}} =$

$= \frac{2}{2\pi} \cdot \left( \frac{1}{1+2k} + \frac{1}{1-2k} \right) = \frac{2}{2\pi} \left( \frac{1-2k+1+2k}{1-4k^2} \right) = \frac{2}{\pi \cdot (1-4k^2)}$   
 $\sin(x+\beta) + \sin(x-\beta) = 2\sin x \cdot \cos \beta$

$b_{2k} = \frac{2}{T} \cdot \int_0^{\frac{T}{2}} \cos t \cdot \sin 2kt dt = \frac{2}{2\pi} \int_0^{\frac{\pi}{2}} \cos t \cdot \sin 2kt dt$

$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin(2k+1)t + \sin(2k-1)t) dt =$

$= \frac{1}{\pi} \left[ -\frac{\cos(2k+1)t}{2k+1} - \frac{\cos(2k-1)t}{2k-1} \right]_0^{\frac{\pi}{2}} =$

$= \frac{1}{\pi} \cdot \left( \frac{1}{2k+1} + \frac{1}{2k-1} \right) = \frac{4k}{\pi \cdot (4k^2-1)}$

$a_k = \frac{-2}{\pi \cdot (4k^2-1)}$

$\Rightarrow f(t) = \frac{1}{\pi} + \sum_{k=1}^{\infty} \frac{-2}{\pi \cdot (4k^2-1)} \left( \cos 2kt + (-1)^k \cdot 2k \sin 2kt \right) \quad t \in \mathbb{R}$

Z.09 d)

$$f(t) = \begin{cases} t & + \varepsilon < 0 < \pi \\ 2\pi - t & + \varepsilon < \pi < 2\pi \end{cases} \quad T = 2\pi \quad \omega = \frac{2\pi}{T} = 1$$

$$a_0 = \frac{2}{T} \int_0^{\pi} t dt + \int_{\pi}^{2\pi} (2\pi - t) dt =$$

$$= \frac{1}{\pi} \left[ t^2 \right]_0^{\pi} + \frac{1}{\pi} \left[ 2\pi t - \frac{1}{2} t^2 \right]_{\pi}^{2\pi} = \frac{1}{\pi} (\pi^2 - 0) + \frac{1}{\pi} (2\pi \cdot 2\pi - \frac{1}{2} (2\pi)^2 - (\frac{1}{2} \pi^2 - \frac{1}{2} \pi^2)) = \frac{3\pi}{2}$$

$$a_{\varepsilon} = \frac{2}{T} \int_0^{\pi} t \cdot \cos \varepsilon t dt + \int_{\pi}^{2\pi} (2\pi - t) \cdot \cos \varepsilon t dt =$$

$$= \frac{1}{\pi} \left( \left[ \frac{t \sin \varepsilon t}{\varepsilon} \right]_0^{\pi} + \left[ \frac{2\pi t \sin \varepsilon t}{\varepsilon} \right]_{\pi}^{2\pi} - \int_{\pi}^{2\pi} t \cdot \cos \varepsilon t dt \right)$$

$$= \frac{1}{\pi} \frac{(-1)^{\varepsilon} - 1}{\varepsilon^2}$$

$$\int_{\pi}^{2\pi} t \cdot \cos \varepsilon t dt \left| \begin{matrix} n = t \cdot \frac{1}{\varepsilon} = \frac{1}{\varepsilon} \cos \varepsilon t \\ m = 1 \cdot \varepsilon = \varepsilon \sin \varepsilon t \end{matrix} \right. = \frac{t \sin \varepsilon t}{\varepsilon} - \frac{1}{\varepsilon} \int \sin \varepsilon t dt =$$

$$= \left[ \frac{t \sin \varepsilon t}{\varepsilon} + \frac{1}{\varepsilon^2} \cos \varepsilon t \right]_{\pi}^{2\pi} = 0 - 0 + \frac{-1 - (-1)^{\varepsilon}}{\varepsilon^2}$$

$$b_{\varepsilon} = \frac{2}{T} \left( \int_0^{\pi} t \cos \varepsilon t dt + \int_{\pi}^{2\pi} (2\pi - t) \sin \varepsilon t dt \right) =$$

$$= \frac{1}{\pi} \int_0^{\pi} t \cos \varepsilon t dt + \frac{1}{\pi} \cdot 2\pi \int_{\pi}^{2\pi} \sin \varepsilon t dt - \frac{1}{\pi} \int_{\pi}^{2\pi} t \sin \varepsilon t dt$$

$$\int_{\pi}^{2\pi} t \sin \varepsilon t dt \left| \begin{matrix} n = t \cdot \varepsilon = \varepsilon t \\ m = 1 \cdot (-1) = -\cos \varepsilon t \end{matrix} \right. = \frac{-t \cos \varepsilon t}{\varepsilon} + \frac{1}{\varepsilon} \int \cos \varepsilon t dt =$$

$$= \left[ \frac{-t \cos \varepsilon t}{\varepsilon} + \frac{1}{\varepsilon^2} \sin \varepsilon t \right]_{\pi}^{2\pi} = \frac{(-2\pi) - (-1)^{\varepsilon} \cdot (-\pi)}{\varepsilon}$$

$$\Rightarrow \left[ \frac{\sin \varepsilon t}{\varepsilon} \right]_0^{\pi} + \left[ \frac{2 \cdot (-\cos \varepsilon t)}{\varepsilon} \right]_{\pi}^{2\pi} + \frac{(-\pi) + (-1)^{\varepsilon} \cdot (\pi)}{\varepsilon} =$$

$$= \frac{2 \cdot (-1 + 1)}{\varepsilon} = 0$$

$$f(t) \sim \frac{3\pi}{4} + \sum_{\varepsilon=1}^{\infty} \left( \frac{(-1)^{\varepsilon} - 1}{\pi \varepsilon^2} \cos \varepsilon t + \frac{1}{\varepsilon} \sin \varepsilon t \right); t \in \mathbb{R}$$