6.11.

a) $f(z) = \dfrac{3z-5}{z^2+2z-3}$; $z_0 = 0$

$$\boxed{\sum_{k=0}^{\infty} \frac{z^k}{a^k} = \frac{a}{a-z} \; ; \; |z| < |a|}$$

$D = 4 + 12 = 16$

$z_{1,2} = \dfrac{-2 \pm 4}{2} \to -3, \; \to 1$

$$= \frac{3z-5}{(z+3)\cdot(z-1)} = \frac{A}{z+3} + \frac{B}{z-1}$$

$$A = \frac{3z-5}{z-1}\bigg|_{z=-3} = \frac{-9-5}{-4} = \frac{-14}{-4} = \boxed{\frac{7}{2}}$$

$$B = \frac{3z-5}{z+3}\bigg|_{z=1} = \frac{3-5}{4} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

$$= \frac{\frac{7}{2}}{z+3} - \frac{\frac{1}{2}}{z-1} =$$

$$\frac{7}{2\cdot 3} \cdot \frac{1}{1-\left(-\frac{z}{3}\right)} + \frac{1}{2} \cdot \frac{1}{1-z} =$$

$$\frac{7}{6} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot (z)^k}{3^k} + \frac{1}{2} \cdot \sum_{k=0}^{\infty} z^k =$$

$$= \frac{1}{2} \cdot \left(\sum_{k=0}^{\infty} \frac{7\cdot(-1)^k \cdot z^k}{3^{k+1}} + \frac{3^{k+1}}{3^{k+1}} \cdot z^k \right) = \quad |z| < \min\{1; 3\}$$

$$\boxed{= \frac{1}{2} \sum_{k=0}^{\infty} \frac{7\cdot(-1)^k + 3^{k+1}}{3^{k+1}} \cdot z^k} \qquad |z| < 1$$

---

b) $f(z) = \dfrac{z}{z^2+3}$; $z_0 = 0$

$$\frac{a}{a-z} = \sum_{k=0}^{\infty} \frac{z^k}{a^k} \quad |z| < |a|$$

$$\frac{z}{z^2+3} = \frac{z}{3} \cdot \frac{1}{1-\left(-\frac{z^2}{3}\right)} = \frac{z}{3} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{2k}}{3^k} = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{2k+1}}{3^{k+1}}}$$

$$\left|-\frac{z^2}{3}\right| < 1 \to \boxed{z < \sqrt{3}}$$

6.11

a) $f(z) = \dfrac{5z-1}{z^2-z-12}$; $z_0 = 1$ $\quad (z_0 - 1 = 0)$

$$\frac{a}{a-z} = \sum_{k=0}^{\infty} \frac{z^k}{a^k}$$

$$D = 1+48 = 49$$

$$z_{12} = \frac{1 \pm 7}{2} \to -4, \; +3$$

$$= \frac{5z-1}{(z+4)\cdot(z-3)} = \frac{A}{z+4} + \frac{B}{z-3}$$

$$A = \left.\frac{5z-1}{z-3}\right|_{z=-4} = \frac{-21}{-7} = 3$$

$$B = \left.\frac{5z-1}{z+4}\right|_{z=3} = \frac{14}{7} = 2$$

$$= \frac{3}{z+4} + \frac{2}{z-3}$$

$$= \frac{3}{(z-1)+1+4} + \frac{2}{(z-1)+1-3} = \frac{3}{(z-1)+5} + \frac{2}{(z-1)-2}$$

$$= \frac{3}{5}\cdot\frac{1}{\left(\frac{z-1}{5}\right)+1} + \frac{2}{2}\frac{\cancel{1}}{\left(\frac{z-1}{2}\right)-1} = \frac{3}{5}\cdot\frac{1}{1-\left(\frac{1-z}{5}\right)} - \frac{1}{1-\left(\frac{z-1}{2}\right)} =$$

$$= \frac{3}{5}\cdot\sum_{k=0}^{\infty}\frac{(1-z)^k}{5^k} - \sum_{k=0}^{\infty}\frac{(z-1)^k}{2^k} = \frac{3}{5}\cdot\sum_{k=0}^{\infty}\frac{(1-z)^k\cdot 2^k}{5^k\cdot 2^k} - \sum_{k=0}^{\infty}\frac{(z-1)^k\cdot 5^k}{2^k\cdot 5^k} =$$

$$= \frac{3}{5}\sum_{k=0}^{\infty}\frac{(-1)^k\cdot(z-1)^k\cdot 2^k}{10^k} - \sum_{k=0}^{\infty}\frac{(z-1)^k\cdot 5^k}{10^k} =$$

$$= \frac{3}{5}\sum_{k=0}^{\infty}\frac{(-2)^k - 5^k}{10^k}\cdot(z-1)^k = \sum_{k=0}^{\infty}\frac{3\cdot(-2)^k - 5^{k+1}}{5\cdot 10^k}\cdot(z-1)^k$$

Patří jen spolu! Zle zapsaný!

6.11 d) $f(z) = \frac{4z-3}{z+3}$; $z_0 = 2-j$ $\qquad (4z-3):(z+3) = 4 - \frac{15}{z+3}$

$-(4z+12)$

$0 - 15$

$z_0 - 2 + j = 0$

$$4 - \frac{15}{z+3} = 4 - \frac{15}{(5-j)+(z_0-2+j)} =$$

$z+3 = z_0 - 2 + j$

$5 - j \neq 0$

$$= 4 - \frac{15}{(5-j)} \cdot \frac{1}{1+\left(\frac{z-2+j}{5-j}\right)} =$$

$$= 4 - \frac{15}{5-j} \cdot \frac{1}{1-\left(-\frac{z-2+j}{5-j}\right)} = 4 - \frac{15}{5-j} \cdot \sum_{k=0}^{\infty} \frac{(z-2+j)^k}{(5-j)^k} =$$

$$= 4 - \frac{15}{5-j} \cdot \frac{5+j}{5+j} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k (z-2+j)^k}{(5-j)^k} \cdot \frac{(5+j)^k}{(5+j)^k} =$$

$$= 4 - \frac{15 \cdot (5+j)}{26} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot (5+j)^k}{26^k} \cdot (z-2+j)^k =$$

$$= \underbrace{\frac{29-15j}{26}}_{k=0} + \underbrace{\sum_{k=1}^{\infty} \frac{15 \cdot (-1)^{k+1} \cdot (5+j)^{k+1}}{26^{k+1}} \cdot (z-2+j)^k}_{\text{začíná } k=1}$$? (řekl bych že k+1 ale výsledek)

$$\left| \frac{z-2+j}{5-j} \right| < 1$$

6.30

a) $\displaystyle\sum_{k=1}^{\infty} \frac{z^k}{k \cdot \ln(k+1)}$; $r = ?$

$$= \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \frac{\dfrac{z^{k+1}}{(k+1)\cdot\ln(k+2)}}{\dfrac{z^k}{k\cdot\ln(k+1)}} = \lim_{k\to\infty} \frac{z^{k+1}}{z^k} \cdot \frac{k\cdot\ln(k+1)}{(k+1)\cdot\ln(k+2)} =$$

$$= \lim_{k\to\infty} z^{k+1-k} \cdot \frac{k\cdot\ln(k+1)}{(k+1)\cdot\ln(k+2)} = |z| \cdot \lim_{k\to\infty} \frac{k\cdot\ln(k+1)}{(k+1)\cdot\ln(k+2)} =$$

$$\Rightarrow \lim_{k\to\infty} \frac{k\cdot\ln(k+1)}{(k+1)\cdot\ln(k+2)} \overset{\text{l'H}}{=} \lim_{k\to\infty} \frac{\ln(k+1) + \dfrac{1}{k+1}}{\ln(k+2) + \dfrac{1}{k+2}} =$$

$$= \lim_{k\to\infty} \frac{\ln\left(1+\frac{1}{k}\right) + \dfrac{1}{\frac{1}{1}+\frac{1}{k}}}{\ln\left(1+\frac{2}{k}\right) + \dfrac{1}{1+\frac{2}{k}}} = \frac{\ln(1+0) + \dfrac{1}{1+0}}{\ln(1+0) + \dfrac{1}{1+0}} =$$

$$= \frac{0+1}{0+1} = 1 \Rightarrow r = 1$$

---

b) $\displaystyle\sum_{k=0}^{\infty} \frac{z^k}{\sqrt{\;}\cdot 2^{k+1}}$; $r = ?$

$$r = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \frac{\dfrac{z^{k+1}}{\sqrt{\;}\cdot 2^{k+2}}}{\dfrac{z^k}{\sqrt{\;}\cdot 2^{k+1}}} = \lim_{k\to\infty} \frac{z^{k+1}}{z^k} \cdot \frac{\sqrt{\;}\cdot 2^{k+2}}{\sqrt{\;}\cdot 2^{k+1}} =$$

$$= \lim_{k\to\infty} z \cdot 2^{k+2-k-1} = z \cdot \lim_{k\to\infty} 2^1 = \boxed{2 = r}$$

6.20

c) $\sum_{k=0}^{\infty} \left(2^k + (-1)^{k+1}\right) z^k \; ; \; r = ?$

$\sum_{k=0}^{\infty} (2z)^k - \sum_{k=0}^{\infty} (-z)^k$ → goniometrická řada

$q_1 = 2z \; ; \; q_2 = -z$

$|z| < \frac{1}{2} \quad |z| < 1 \Rightarrow r = \min\{r_1; r_2\} =$

$= \min\{\frac{1}{2}; 1\} \Rightarrow r = \frac{1}{2}$

---

d) $\sum_{k=0}^{\infty} \frac{z^{2k+1}}{\sqrt{k+1}} \; ; \; r = ?$

$A = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\frac{z^{2k+3}}{\sqrt{k+2}}}{\frac{z^{2k+1}}{\sqrt{k+1}}} = \lim_{k\to\infty} \frac{z^{2k+3}}{z^{2k+1}} \cdot \frac{\sqrt{k+1}}{\sqrt{k+2}} =$

$= \lim_{k\to\infty} z^{2k+3-2k-1} \cdot \frac{\sqrt{1+\frac{1}{k}}}{\sqrt{1+\frac{2}{k}}} = |z^2| \cdot \lim_{k\to\infty} \frac{\sqrt{1+0}}{\sqrt{1+0}} = \boxed{1}$

$\boxed{r = 1}$

6.31

a) $\displaystyle\sum_{k=0}^{\infty} \frac{(z-1)^{2k}}{(k+1)^2}$; r; obor

$$A = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\frac{(z-1)^{2k+2}}{(k+2)^2}}{\frac{(z-1)^{2k+1}}{(k+1)^2}} = \lim_{k\to\infty} (z-1)^{\overset{1}{\overbrace{2k+2-2k-1}}} \cdot \frac{(k+2)^2}{(k+1)^2} =$$

$$= (z-1) \cdot \lim_{k\to\infty} \frac{(k+2)^2}{(k+1)^2} = (z-1) \cdot \lim_{k\to\infty} \frac{k^2+4k+4}{k^2+2k+1} \overset{(l'H)^2}{=}$$

$$= (z-1) \cdot \lim_{k\to\infty} \frac{2}{2} = 1 \Rightarrow r = 1$$

$$\text{obor } |z-1| \leq 1$$

---

b) $\displaystyle\sum_{k=2}^{\infty} \frac{(z+j)^k}{3^k}$, r; obor ?

$$A = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\frac{(z+j)^{k+1}}{3^{k+1}}}{\frac{(z+j)^k}{3^k}} = \lim_{k\to\infty} \frac{(z+j)^{k+1}}{(z+j)^k} \cdot \frac{3^k}{3^{k+1}} =$$

$$= \lim_{k\to\infty} (z+j)^{k+1-k} \cdot 3^{k-k-1} = (z+j) \lim_{k\to\infty} 3^{-1} \Rightarrow$$

$$\frac{|z+j|}{3} < 1$$

$$\boxed{|z+j| < 3 \Rightarrow r = 3 \text{ ?}}$$

6.31.

a) $\sum_{k=1}^{\infty} \left(\frac{z-1}{2j+1}\right)^{3k} \to r;$ obor ?

$$A = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{\left(\frac{z-1}{2j+1}\right)^{3k+1}}{\left(\frac{z-1}{2j+1}\right)^{3k}} = \left(\frac{z-1}{2j+1}\right) \cdot \lim_{k\to\infty} 1$$

$$\left|\frac{z-1}{2j+1}\right| < 1 \Rightarrow |z-1| < |2j+1| = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\underbrace{|z-1| < \sqrt{5}}_{} \qquad \underbrace{r = \sqrt{5}}_{}$$

obor $\longrightarrow$

6.32. Urči T. Radu

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!} \qquad x \in R$$

a) $f(x) = \frac{1-\cos x}{x^2}$; $f(0) = \frac{1}{2}$

$$f(x) = \frac{1}{x^2}\left(1 - \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!}\right) = \frac{1}{x^2} \cdot \left(1 + (-1) \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!}\right) =$$

$$= \frac{1}{x^2}\left(1 + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot x^{2k}}{(2k)!}\right) = x^{-2} \cdot \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^{2k}}{(2k)!} =$$

$k=0$ je suma $= -1$

$1 + -1 = 0 \Rightarrow$ začneme $k = 1$ °

$$= \underbrace{\sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^{2(k-1)}}{(2k)!}}_{}$$

6.32 b) $f(x) = \dfrac{e^{2x}-1}{x}$; $f(0) = 2$ $\qquad e^x = \sum\limits_{k=0}^{\infty} \dfrac{x^k}{k!} \quad x \in \mathbb{R}$

$$f(x) = \frac{1}{x}\cdot\left(\sum_{k=0}^{\infty} \frac{2^k x^k}{k!} - 1\right) = \frac{1}{x}\cdot\sum_{k=1}^{\infty} \frac{2^k \cdot x^k}{k!} =$$

$$= \sum_{k=1}^{\infty} \frac{2^k \cdot x^{k-1}}{k!}$$

---

c) $f(x) = \sin^2 x = \dfrac{1}{2}\left(1 - \cos 2x\right)$ $\qquad \cos x = \sum\limits_{k=0}^{\infty} \dfrac{(-1)^k x^{2k}}{(2k)!}$

$$f(x) = \frac{1}{2}\left(1 - \sum_{k=0}^{\infty} \frac{(-1)^k (2x)^{2k}}{(2k)!}\right) = \frac{1}{2}\cdot\left(\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\cdot(2x)^{2k}}{(2k)!}\right) =$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\cdot 2^{(2k-1)}\cdot x^{2k}}{(2k)!}$$

---

d) $\sin x \cdot \cos 3x =$; $x \in \mathbb{R}$

$x = \alpha$
$3x = \beta$

$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha$

$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha$

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\cdot\sin\alpha\cdot\cos\beta$

$\dfrac{\sin(x+3x) + \sin(x-3x)}{2} = \sin x\cdot\cos 3x$

$\dfrac{1}{2}\left(\sin 4x + \sin(-2x)\right) = \sin x\cdot\cos 3x$

$\dfrac{1}{2}\left(\sin 4x - \sin 2x\right) = \sin x\cdot\cos 3x$

$\sin x = \sum\limits_{k=0}^{\infty} \dfrac{(-1)^k x^{2k+1}}{(2k+1)!} \quad x \in \mathbb{R}$

$\cos 3x = \sum\limits_{k=0}^{\infty} \dfrac{(-1)^k \cdot x^{2k}}{(2k)!}$

$$\sin x\cdot\cos 3x = \frac{1}{2}\left(\sin 4x - \sin 2x\right) = \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{(-1)^k\cdot(4x)^{2k+1}}{(2k+1)!} - 1\cdot\sum_{k=0}^{\infty} \frac{(-1)^k\cdot(2x)^{2k+1}}{(2k+1)}\right) =$$

$$= \cancel{\frac{1}{2}}\left(\sum_{k=0}^{\infty} \frac{(-1)^k\cdot 4^{2k}\cdot 4\cdot x^{2k+1}}{(2k+1)!} - \sum_{k=0}^{\infty} \frac{(-1)^k\cdot 2^{2k}\cdot \cancel{2}\cdot x^{2k+1}}{(2k+1)!}\right) =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k\cdot 4^k\cdot(2\cdot 4^k - 1)}{(2k+1)!}\, x^{2k+1}$$

6.32

e) $\sinh x \; ; x \in \mathbb{R} \qquad \sinh x = \frac{1}{2}(e^{x} - e^{-x}) \quad ; e^{x} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$\sinh x = \frac{1}{2} \cdot \left(\sum_{k=0}^{\infty} \frac{x^k}{k!} - \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!} \right) =$$

$$= \frac{1}{2} \left(\sum_{k=0}^{\infty} \frac{x^k}{k!} + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot x}{k!} \right) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{x^{2k+1}}{k!} =$$

$$= \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k)!}$$

---

f) $\cosh x = \frac{1}{2}(e^{x} + e^{-x}) \quad ; \quad e^{x} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$\cosh x = \frac{1}{2}(e^{x} + e^{-x}) = \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{x^k}{k!} + \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!} \right) =$$

$$= \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

6.33. $f(x) = \int_0^x \frac{\sin t}{t} \, dt = \qquad \sin t = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{(2k+1)!}$

$$= \int_0^x \frac{1}{t} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^{2k+1}}{(2k+1)!} = \int_0^x \sum_{k=0}^{\infty} \frac{(-1) \cdot t^{2k}}{(2k+1)!} \, dt =$$

$$= \sum_{k=0}^{\infty} \int_0^x \frac{(-1)^k \cdot t^{2k}}{(2k+1)!} \cdot dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot \left[\frac{t^{2k+1}}{2k+1} \right]_0^x =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^{2k+1}}{(2k+1)! \cdot (2k+1)} \; ; x \in \mathbb{R}$$

6.77.

$$f(x) = \int_0^x \frac{1-e^{-t}}{t}\, dt = \int_0^x \frac{1}{t}\cdot\left(1-e^{-t}\right) dt =$$

$$= \int_0^x \frac{1}{t}\cdot\left(1-\sum_{k=0}^{\infty}\frac{(-1)^k\cdot t^k}{k!}\right)dt = \int_0^x t^{-1}\cdot\left(1+\sum_{k=0}^{\infty}\frac{(-1)^{k+1}\cdot t^{k}}{k!}\right)dt =$$

$$= \int_0^x t^{-1}\cdot\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cdot t^k}{k!} = \sum_{k=1}^{\infty}\int_0^x \frac{(-1)^{k+1}}{k!}\cdot t^{k-1}\, dt =$$

$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}\cdot\left[\frac{t^k}{(k-1)+1}\right]_0^x = \underline{\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cdot x^k}{k\cdot k!}}$$

---

c) $$f(x) = \int_0^x (1+t^4)^{-\frac{1}{2}}\, dt = \qquad (1+x)^n = \sum_{k=0}^{\infty}\binom{n}{k}x^k.$$

$$= \int_0^x \sum_{k=0}^{\infty}\binom{-\frac{1}{2}}{k}t^{4k} =$$

$$= \sum_{k=0}^{\infty}\binom{-\frac{1}{2}}{k}\cdot\int_0^x t^{4k} = \sum_{k=0}^{\infty}\binom{-\frac{1}{2}}{k}\cdot\left[\frac{t^{4k+1}}{4k+1}\right]_0^x =$$

$$= \underline{\sum_{k=0}^{\infty}\binom{-\frac{1}{2}}{k}\cdot\frac{x^{4k+1}}{4k+1}} = x + \sum_{k=1}^{\infty}\frac{(-1)^k\cdot(2k-1)!!}{(2k)!!\cdot(4k+1)} \quad ?$$

F. ŘAD^y

7.14. a) $f(t) = \cos t \; ; t \in \langle 0; \frac{\pi}{2})$ $\Rightarrow T = \frac{\pi}{2} \; ; \; \omega = \frac{2\pi}{T} = \boxed{4}$

$$a_0 = \frac{2}{T} \cdot \int_a^{a+T} f(x)\, dx$$

$$a_0 = \frac{2}{\frac{\pi}{2}} \cdot \int_0^{\frac{\pi}{2}} \cos t \, dt = \frac{4}{\pi} \cdot \Big[\sin t\Big]_0^{\frac{\pi}{2}} = \frac{4}{\pi} \cdot 1 = \boxed{\frac{4}{\pi}}$$

$$a_k = \frac{2}{T} \cdot \int_\alpha^{\alpha+T} f(t) \cdot \cos k \cdot \omega t \, dt =$$

$$\Rightarrow \frac{2}{\frac{\pi}{2}} \cdot \int_0^{\frac{\pi}{2}} \cos t \cdot \cos 4kt \, dt \Rightarrow$$

$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$ $\quad \alpha = 4kt$

$\cos(\alpha-\beta) = \text{—} \;\; \text{''} \;\; \text{—} \; + \; \text{—} \;\; \text{''} \;\; \text{——}$ $\quad \beta = t$

$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2 \cdot \cos\alpha \cdot \cos\beta \Rightarrow$

$\cos t \cdot \cos 4kt = \frac{1}{2} \cdot \Big[\cos(4kt+t) + \cos(4kt-t)\Big]$

$$= \frac{1}{2} \frac{4}{\pi} \cdot \int_0^{\frac{\pi}{2}} \Big(\cos\big[(4k+1) \cdot t\big] + \cos\big[(4k-1)t\big]\Big) dt =$$

$$= \frac{2}{\pi} \cdot \left[\frac{\sin\big[(4k+1)t\big]}{4k+1} + \frac{\sin\big[(4k-1)t\big]}{4k-1}\right]_0^{\frac{\pi}{2}} =$$

$$= \frac{2}{\pi} \cdot \left[\frac{\sin(4k+1)\frac{\pi}{2}}{4k+1} - \frac{\sin(4k-1)\frac{\pi}{2}}{4k-1}\right] =$$

$\sin\left(\frac{8k}{2}\pi + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$

Periody!

$$= \frac{2}{\pi} \left[\frac{1}{4k+1} - \frac{1}{4k-1}\right] = \frac{2}{\pi} \cdot \left(\frac{4k-1-4k-1}{16k^2-1}\right) = \boxed{\frac{-4}{\pi \cdot (16k^2-1)}}$$

$\Longrightarrow$

$$b_k = \frac{2}{T} \cdot \int_{\alpha}^{\alpha+T} f(t) \cdot \sin k\omega t \, dt =$$

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$

$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha \cdot \cos\beta$

$\alpha = 4kt \; ; \; \beta = t$

$$= \frac{2}{\frac{\pi}{2}} \cdot \int_0^{\frac{\pi}{2}} \cos t \cdot \sin 4kt \, dt =$$

$$= \frac{1}{2} \cdot \frac{4}{\pi} \cdot \int_0^{\frac{\pi}{2}} \left[\sin(4kt+t) + \sin(4kt-t) \right] dt =$$

$$= \frac{2}{\pi} \cdot \left[-\frac{\cos[(4k+1)t]}{4k+1} - \frac{\cos[(4k-1)t]}{4k-1} \right]_0^{\frac{\pi}{2}} =$$

$$= -\frac{2}{\pi} \cdot \left[\frac{\cos\left(\frac{4k\pi}{2} + \frac{\pi}{2}\right)}{4k+1} + \frac{\cos\left(\frac{4k\pi}{2} - \frac{\pi}{2}\right)}{4k-1} \right] -$$

$$- \frac{2}{\pi} \cdot \left[\frac{-\cos 0}{4k+1} - \frac{\cos 0}{4k-1} \right] = 0 + \frac{2}{\pi} \left(\frac{1}{4k+1} + \frac{1}{4k-1} \right) =$$

$$= \frac{2}{\pi} \left(\frac{4k-1+4k+1}{16k^2-1} \right) = \frac{2 \cdot 8k}{\pi \cdot (16k^2-1)} = \boxed{\frac{16k}{\pi \cdot (16k^2-1)}}$$

F.R.

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cdot \cos k\omega t + b_k \cdot \sin k\omega t \right) =$$

$$= \frac{2}{\pi} + \sum_{k=1}^{\infty} \left(-\frac{4}{\pi \cdot (16k^2-1)} \cdot \cos 4kt + \frac{16k}{\pi \cdot (16k^2-1)} \cdot \sin 4kt \right) =$$

$$= \boxed{\frac{2}{\pi} + \sum_{k=1}^{\infty} \frac{4}{\pi \cdot (16k^2-1)} \cdot \left(4k \sin 4kt - \cos 4kt \right)}$$

7.14 b) $f(t) = t^2 \quad t \in \langle -1; 1 \rangle \qquad T = 2 \; ; \; \omega = \frac{2\pi}{T} = \pi$

$$a_0 = \frac{2}{T} \cdot \int_{\alpha}^{\alpha+T} t^2\,dt = 1 \cdot \int_{-1}^{1} \frac{1}{3} t^3 = \left[\frac{1}{3} t^3 \right]_{-1}^{1} = \frac{1}{3} - \frac{(-1)}{3} = \boxed{\frac{2}{3}}\, a_0$$

$$a_k = \frac{2}{T} \int_{\alpha}^{\alpha+T} t^2 \cdot \cos k\omega t\,dt = \int_{-1}^{1} t^2 \cdot \cos k\pi t\,dt = \left| \begin{matrix} u = t^2 & v' = \cos k\pi t \\ u' = 2t & v = \frac{\sin k\pi t}{k\pi} \end{matrix} \right|$$

$$= t^2 \cdot \frac{\sin k\pi t}{k\pi} \Big|_{-1}^{1} - \frac{2}{k\pi} \int_{-1}^{1} t \cdot \sin k\pi t\,dt = \left| \begin{matrix} u = t & v' = \sin k\pi t \\ u' = 1 & v = -\frac{\cos k\pi t}{k\pi} \end{matrix} \right| =$$

$$= \frac{t^2 \cdot \sin k\pi t}{k\pi} \Big|_{-1}^{1} + \frac{2}{k\pi} \cdot \left[\frac{t \cdot \cos k\pi t}{k\pi} \Big|_{-1}^{1} + \frac{1}{k\pi} \int_{-1}^{1} 1 \cdot \cos k\pi t\,dt \right]$$

$$= \left[\frac{t^2 \cdot \sin k\pi t}{k\pi} + \frac{2}{k\pi} \left[\frac{t \cdot \cos k\pi t}{k\pi} + \frac{1}{k\pi} \cdot \frac{\sin k\pi t}{k\pi} \right] \right]_{-1}^{1} =$$

$$= \left(\frac{t^2 \cdot \sin k\pi t}{k\pi} + 2t \cdot \frac{\cos k\pi t}{k^2 \cdot \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin k\pi t \right]_{-1}^{1} =$$

$$= \frac{1 \cdot \sin k\pi}{k\pi} + \frac{2 \cdot \cos k\pi}{k^2 \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin k\pi -$$

$$\left[\frac{1 \cdot \sin(-k\pi)}{k\pi} - \frac{2 \cdot \cos(-k\pi)}{k^2 \pi^2} + \frac{2}{k^3 \pi^3} \cdot \sin(-k\pi) \right] =$$

$$= 0 + \frac{2 \cdot (-1)}{k^2 \pi^2} + 0 - \left(\frac{0}{k\pi} - \frac{2 \cdot (-1)}{k^2 \pi^2} + 0 \right) =$$

$$= \frac{-4}{k^2 \pi^2} \Rightarrow \frac{(-1)^k \cdot 4}{k^2 \cdot \pi^2}$$

$\Rightarrow$

$$b_k = \frac{2}{T}\int_{\alpha}^{\alpha+T} t^2 \cdot \sin k\omega t\,dt =$$

vis gradahon!

$$= \int_{-1}^{1} t^2 \cdot \sin k\pi t \Rightarrow$$

$$\left[t^2 \frac{(-\cos k\pi t)}{k\pi} + 2t \cdot \frac{(\sin k\pi t)}{k^2\pi^2} + \frac{2}{k^3\pi^3} \cdot (-\cos k\pi t)\right]_{-1}^{1} =$$

$$= \left(\frac{t^2 \cdot \cos k\pi}{k\pi} + 0 + \frac{2}{k^3\pi^3} \cdot (-\cos k\pi)\right) -$$

$$-\left(\frac{t_1^2}{k\pi} \cdot 1 + 0 + \frac{2}{k^3\pi^3} \cdot 1\right) = 0 = b_k$$

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos\omega t + b_k \cdot \sin\omega t =$$

$$= \frac{1}{3} + \sum_{k=1}^{\infty} \frac{(-1)^k \cdot 4}{k^2 \cdot \pi^2} \cdot \cos k\pi t$$

7.38

a) $f(t)=3\sin^2 t+2\cos^2 t=3\cdot\frac{1}{2}\left(1-\cos 2t\right)+\frac{2}{2}\cdot\left(1+\cos 2t\right)=$

$=\frac{3}{2}-\frac{3}{2}\cos 2t+1+\cos 2t=\underbrace{\frac{5}{2}-\frac{1}{2}\cdot\cos 2t}$

b) $f(t)=5\cdot\cos^3 t=5\cdot\cos\cdot\frac{1}{2}\cdot\left(1+\cos 2t\right)=\frac{5}{2}\cos t+\frac{5}{2}\cos t\cdot\cos 2t=$

$=\frac{5}{2}\cos t+\frac{5}{2}\cdot\left(\frac{\cos 3t+\cos(t)}{2}\right)=$

$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cdot\cos\beta$

$=\frac{5}{2}\cos t+\frac{5}{4}\cos 3t+\frac{5}{4}\cos(t)=\frac{15}{4}\cos t+\frac{5}{4}\cos 3t$

c) $f(t)=7\cdot\sin^4 t$

$\sin^4 t=\frac{(1-\cos 2t)}{2}\cdot\frac{(1-\cos 2t)}{2}=\frac{1}{4}\left(1-2\cos 2t+\cos^2 2t\right)=$

$=\frac{1}{4}\left(1-2\cos 2t+\frac{1}{2}\cdot\left(1+\cos 4t\right)\right)=\frac{1}{4}\left(1-2\cos 2t+\frac{1}{2}+\frac{1}{2}\cos 4t\right)=$

$=\frac{1}{4}-\frac{1}{2}\cos 2t+\frac{1}{8}+\frac{1}{8}\cos 4t=\frac{3}{8}-\frac{1}{2}\cos 2t+\frac{1}{8}\cdot\cos 4t$

$\Rightarrow 7\cdot\sin^4 t=\frac{7\cdot 3}{8}-7\cdot\frac{1}{2}\cos 2t+7\cdot\frac{1}{8}\cos 4t=\underbrace{\frac{21}{8}-\frac{7}{2}\cos 2t+\frac{7}{8}\cos 4t}$

d) $5\sin t\cdot\cos 3t=$

$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cdot\cos\beta$

$=\frac{5}{2}\left(\sin 4t+\sin(-2t)\right)=\underbrace{\frac{5}{2}\sin 4t-\frac{5}{2}\sin 2t}$

e) $f(t)=8\sin 3t\cdot\sin 2t=$

$-\cos(\alpha+\beta)=-\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta$

$\cos(\alpha-\beta)-\cos(\alpha+\beta)=2\sin\alpha\cdot\sin\beta$

$=\frac{8}{2}\left(\cos(t)-\cos(5t)\right)=$

$=\underbrace{4\cos t-4\cos 5t}$

$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cdot\cos\beta$

f) $15\cos 4t\cdot\cos t=\underbrace{\frac{15}{2}\left(\cos 5t+\cos 3t\right)}$

7.14

c) $f(t) = t \quad t \in (-\pi; \pi) \qquad T = 2\pi \qquad k = \frac{2\pi}{T} = 1$

$$a_0 = \int_{\alpha}^{\alpha+T} f(t)\,dt = \int_{-\pi}^{\pi} t\,dt = \left[\frac{1}{2}t^2\right]_{-\pi}^{\pi} = \frac{1}{2}\pi^2 - \frac{1}{2}(-\pi)^2 = \boxed{0}$$

$$a_k = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\cdot\cos kt\,dt = \left| \begin{array}{ll} u = t & v' = \cos kt \\ u' = 1 & v = \frac{\sin kt}{k} \end{array} \right] =$$

$$= \frac{1}{\pi}\left[t\cdot\frac{\sin kt}{k} - \frac{1}{k}\int_{-\pi}^{\pi}\sin kt\,dt\right] =$$

$$= \frac{1}{\pi}\left[t\cdot\frac{\sin kt}{k} + \frac{1}{k^2}\cos kt\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left(0 + \frac{(-1)}{k^2} + 0 - \frac{(-1)}{k^2}\right) = \boxed{0}$$

$$b_k = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\cdot\sin kt\,dt = \frac{1}{\pi}\left[\frac{t\cdot(-\cos kt)}{k} + \frac{1}{k^2}\cdot\sin kt\right]_{-\pi}^{\pi} =$$

$$= \frac{1}{\cancel{\pi}}\left(\frac{\cancel{\pi}(-\cos k\pi)}{k} - \frac{(-\cancel{\pi})}{k}\cdot(-\cos(-\pi k))\right) =$$

$$= \frac{(-\cos k\pi)}{k} + \frac{\cos(-\pi k)}{k} = \frac{-2\cos(k\pi)}{k}$$

$\frac{3}{2}(1 -$

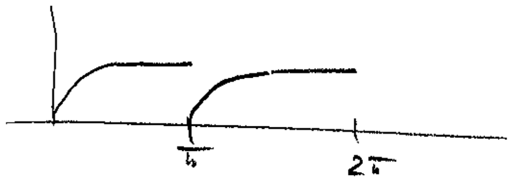
$3\sin^2 t + 2\cdot(1 - \sin^2 t) = 2 + \sin^2 t = 2 + \frac{1}{2}(1 - \cos 2x) =$

$\sqrt{}\cdot\cos t\left(\frac{1}{2} + \cos 2x\right)^2 = \frac{\sqrt{5}}{2} - \frac{\cos 2t}{2}$

$=$

7.39

a) $f(t) = \begin{cases} \sin t & t \in \langle 0; \frac{\pi}{2} \rangle \\ 1 & t \in \langle \frac{\pi}{2}; \pi \rangle \end{cases}$



$T = \pi$; $\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

$$a_k = \frac{2}{T} \cdot \int_0^T f(t) \cdot \cos k\omega t \, dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \cdot \sin k\omega t \, dt$$

FR. $\boxed{\frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cdot \cos k\omega t + b_k \cdot \sin k\omega t \right)}$

$$a_0 = \frac{2}{T}\left(\int_0^{\frac{\pi}{2}} \sin t \, dt + \int_{\frac{\pi}{2}}^{\pi} 1 \, dt \right) = \frac{2}{\pi}\left(\left[-\cos \right]_0^{\frac{\pi}{2}} + \left[t \right]_{\frac{\pi}{2}}^{\pi} \right) =$$

$$= \frac{2}{\pi} \cdot \left(-\cos\frac{\pi}{2} + \cos 0 + \pi - \frac{\pi}{2} \right) = \frac{2}{\pi} \cdot \left(0 + 1 + \pi - \frac{\pi}{2} \right) = \frac{2}{\pi}\left(1 + \frac{\pi}{2} \right)$$

$$= \frac{2}{\pi} + \frac{2\pi}{2\pi} = \boxed{\frac{2+\pi}{\pi}}$$

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha \cdot \cos\beta$

$$a_k = \frac{2}{\pi} \cdot \int_0^{\frac{\pi}{2}} \sin t \cdot \cos 2kt + \int_{\frac{\pi}{2}}^{\pi} \cdot \cos 2kt \, dt$$

$$= \frac{2}{\pi} \cdot \left(\frac{1}{2} \int_0^{\frac{\pi}{2}} \sin(2k+1)t + \sin(1-2k)t \, dt + \int_{\frac{\pi}{2}}^{\pi} \cos 2kt \, dt \right) =$$

$$= \frac{2}{\pi} \cdot \left(\frac{1}{2} \cdot \left[\frac{-\cos(2k+1)t}{2k+1} - \frac{\cos(1-2k)t}{1-2k} \right]_0^{\frac{\pi}{2}} + \left[\frac{\sin 2kt}{2k} \right]_{\frac{\pi}{2}}^{\pi} \right) =$$

$$= \frac{2}{\pi} \cdot \left[\frac{1}{2} \left[\left(\frac{-0}{2k+1} + \frac{-1}{2k+1} \right) + \left(\frac{0}{1-2k} + \frac{-1}{1-2k} \right) \right] + \left(-\frac{0-0}{2k} \right) \right] =$$

$$= \frac{2}{\pi} \cdot \left[\frac{1}{2} \left(\frac{-1}{2k+1} + \frac{-1}{1-2k} \right) \right] = \frac{1}{\pi} \cdot \frac{-1+2k-2k-1}{1-4k^2} = \boxed{-\frac{2}{\pi \cdot (1-4k^2)}}$$

$\Longrightarrow$

7.39 pokr.

$-\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha \sin\beta$

$$b_k = \frac{2}{\pi}\left[\int_0^{\frac{\pi}{2}} \sin t \cdot \sin 2kt + \int_{\frac{\pi}{2}}^{\pi} \sin 2kt \, dt\right] =$$

$$= \frac{2}{\pi}\left[\int_0^{\frac{\pi}{2}} (\sin(1-2k)t - \sin(1+2k)t)\,dt + \int_{\frac{\pi}{2}}^{\pi} \sin 2kt \, dt\right] =$$

$$= \frac{2}{\pi}\left(\left[\frac{-\cos(1-2k)t}{1-2k} + \frac{+\cos(1+2k)t}{1+2k}\right]_0^{\frac{\pi}{2}} + \left[\frac{-\cos 2kt}{2k}\right]_{\frac{\pi}{2}}^{\pi}\right) =$$

$$= \frac{2}{\pi}\frac{1}{2}\left(\frac{+1}{1-2k} - \frac{1}{1+2k}\right) +2\left(\frac{-1}{2k} - \frac{1}{2k}\right) =$$

$$= \frac{2}{\pi}\frac{1}{2}\left[\left(\frac{1+2k-1+2k}{1-4k^2}\right) - 2\cdot\frac{\cancel{2}}{\cancel{2}k}\right] = \frac{2}{2\pi}\cdot\left(\frac{4k}{1-4k^2} - \frac{2\cdot 1}{k}\right) =$$

$$= \boxed{\frac{4}{\pi}\cdot\frac{k}{1-4k^2} - \frac{2}{\pi k}}$$

osciluje podle k?

Rada

?

$$\boxed{f(t) = \frac{2+\pi}{2\pi} + \sum_{k=1}^{\infty} \frac{-2}{\pi\cdot(1-4k^2)}\cdot\cos 2kt + \left(\frac{4k(-1)^{k+1}}{\pi(1-4k^2)} + \frac{1-(-1)^k}{\pi k}\right)\sin 2kt}$$

$t \in R$

7.19 b)

$$f(t) = \begin{cases} \sin t & t \in \langle 0 ; \frac{\pi}{2} \rangle \\ 0 & t \in \langle \frac{\pi}{2} ; \pi \rangle \end{cases} \Rightarrow T = \pi \quad , k = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$a_0 = \frac{2}{T} \int_0^{\frac{\pi}{2}} \sin t \, dt + \int_{\frac{\pi}{2}}^{\pi} 0 \, dt = \frac{2}{\pi} \left[-\cos t \right]_0^{\pi} = \boxed{\frac{2}{\pi}}$$

$$b_k = \frac{2}{\pi} \cdot \int_0^{\frac{\pi}{2}} \sin t \cdot \sin 2kt \, dt =$$

$$-\cos(\alpha+\beta) = -\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2 \sin\alpha \cdot \sin\beta$$

$$= \frac{2}{\pi} \left[\frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(1-2k)t - \cos(1+2k)t \, dt \right] =$$

$$= \frac{1}{\pi} \cdot \left[\frac{\sin(1-2k)t}{1-2k} - \frac{\sin(1-2k)t}{1+2k} \right]_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{\pi} \cdot \left(\frac{1}{1-2k} - \frac{1}{1+2k} \right) = \frac{1}{\pi} \cdot \left(\frac{1+2k-1+2k}{1-4k^2} \right) = \boxed{\frac{4k}{\pi \cdot (1-4k^2)}}$$

$$a_k = \frac{2}{\pi} \cdot \int_0^{\frac{\pi}{2}} \sin t \cdot \cos 2kt \, dt =$$

$$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2 \sin\alpha \cdot \cos\beta$$

$$= \frac{1}{\pi} \cdot \int_0^{\frac{\pi}{2}} \sin(1+2k)t + \sin(1-2k)t \, dt =$$

$$= \frac{1}{\pi} \cdot \left[\frac{-\cos(1+2k)t}{1+2k} - \frac{\cos(1+2k)t}{1-2k} \right]_0^{\frac{\pi}{2}} = \frac{1}{\pi} \left(\frac{-1}{1+2k} + \frac{-1}{1-2k} \right) =$$

$$= \frac{-2}{\pi \cdot (1-4k^2)}$$

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos k\omega t + b_k \cdot \sin k\omega t =$$

$$= \frac{1}{\pi} + \sum_{k=1}^{\infty} \frac{4k}{\pi (1-4k^2)} \sin 2kt + \frac{2}{\pi \cdot (1-4k^2)} \cos 2kt$$

$$= \boxed{\frac{1}{\pi} + \sum_{k=1}^{\infty} \left(\frac{(-1)^k - 1}{\pi \cdot (1-4k^2)} + \frac{4k}{\pi \cdot (1-4k^2)} \cdot \sin 2kt \right) \quad t \in R}$$

7.19

c) $f(t) = \begin{cases} \cos t & t \in \langle 0; \frac{\pi}{2} \rangle \\ 0 & t \in \langle \frac{\pi}{2}; \pi \rangle \end{cases} \Rightarrow T = \pi \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

$$a_0 = \frac{2}{T} \cdot \int_0^{\frac{\pi}{2}} \cos t \, dt = \frac{2}{\pi} \cdot \Big[\sin t \Big]_0^{\frac{\pi}{2}} = \boxed{\frac{2}{\pi}}$$

$$a_k = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos t \cdot \cos 2kt \, dt =$$

$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha \cdot \cos\beta$

$$= \frac{2}{2\pi} \int_0^{\frac{\pi}{2}} \left(\cos(1+2k)t + \cos(1-2k)t \right) dt =$$

$$= \frac{2}{2\pi} \left[\frac{\sin(1+2k)t}{1+2k} + \frac{\sin(1-2k)t}{1+2k} \right]_0^{\frac{\pi}{2}} =$$

$$= \frac{2}{2\pi} \cdot \left(\frac{1}{1+2k} + \frac{1}{1-2k} \right) = \frac{2}{2\pi} \left(\frac{1-2k+1+2k}{1-4k^2} \right) = \boxed{\frac{2}{\pi \cdot (1-4k^2)}}$$

$$b_k = \frac{2}{T} \cdot \int_0^{\frac{\pi}{2}} \cos t \cdot \sin 2kt \, dt =$$

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha \cdot \cos\beta$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\sin(2k+1)t + \sin(2k-1)t \right) dt =$$

$$= \frac{1}{\pi} \cdot \left[-\frac{\cos(2k+1)t}{2k+1} - \frac{\cos(2k-1)t}{2k-1} \right]_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{\pi} \cdot \left(\frac{1}{2k+1} + \frac{1}{2k-1} \right) = \boxed{\frac{4k}{\pi \cdot (4k^2-1)}}$$

$$a_k = -\frac{2}{\pi \cdot (4k^2-1)}$$

$$\Rightarrow \boxed{f(t) = \frac{1}{\pi} + \sum_{k=1}^{\infty} \frac{-2}{\pi \cdot (4k^2-1)} \left(\cos 2kt + (-1)^k \cdot 2k \sin 2kt \right) \quad t \in \mathbb{R}}$$

7.09 d)

$$f(t) = \begin{cases} \pi & t \in \langle 0;\pi) \\ 2\pi - t & t \in \langle \pi; 2\pi) \end{cases} \qquad T = 2\pi \qquad \omega = \frac{2\pi}{T} = 1$$

$$a_0 = \frac{2}{T} \int_0^{\pi} \pi\, dt + \int_{\pi}^{2\pi} (2\pi - t)\, dt =$$

$$= \frac{1}{\pi}\Big[\pi t\Big]_0^{\pi} + \frac{1}{\pi}\left(\Big[2\pi t\Big]_{\pi}^{2\pi} - \left[\frac{1}{2}t^2\right]_{\pi}^{2\pi}\right) = \frac{1}{\pi}(\pi - 0) + \frac{1}{\pi}(2\pi - \pi) -$$

$$-\left(\frac{1}{2}4\pi^2 - \frac{1}{2}\pi^2\right) = 2\pi^2 + \cancel{2\pi^2} - \cancel{2\pi^2} + \frac{\pi^2}{2} = \boxed{\frac{3}{2}\pi}$$

$$a_k = \frac{2}{T} \int_0^{\pi} \pi \cdot \cos kt\, dt + \int_{\pi}^{2\pi} (2\pi - t) \cdot \cos kt\, dt =$$

$$= \frac{1}{\pi}\pi\left(\left[\frac{\sin kt}{k}\right]_0^{\pi} + \underbrace{\left[2\pi \cdot \frac{\sin kt}{k}\right]_{\pi}^{2\pi}}_{0} - \int_{\pi}^{2\pi} t \cdot \cos kt\, dt\right)$$

$$= \boxed{\frac{1}{\pi} \cdot \frac{(-1)^k - 1}{k^2}}$$

$$\int_{\pi}^{2\pi} t \cdot \cos kt\, dt \left| \begin{matrix} u = t & v' = \cos kt \\ u' = 1 & v = \frac{\sin kt}{k} \end{matrix} \right| = \frac{\sin kt}{k} - \frac{1}{k}\int \sin kt\, dt =$$

$$= \left[\frac{\sin kt}{k} + \frac{1}{k^2}\cos kt\right]_{\pi}^{2\pi} = 0 - 0 + \frac{-1 - (1)^k}{k^2}$$

$$b_k = \frac{2}{T}\left(\int_0^{\pi} \pi \cos kt\, dt + \int_{\pi}^{2\pi} (2\pi - t) \cdot \sin kt\, dt\right) =$$

$$= \frac{1}{\pi}\pi \cdot \int_0^{\pi} \cos kt\, dt + \frac{1}{\pi} 2\pi \int_{\pi}^{2\pi} \sin kt - \frac{1}{\pi}\int_{\pi}^{2\pi} t \cdot \sin kt\, dt$$

$$\boxed{\int_{\pi}^{2\pi} t \cdot \sin kt\, dt =} \left| \begin{matrix} u = t & v' = \sin kt \\ u' = 1 & v = -\frac{\cos kt}{k} \end{matrix} \right| = \frac{-t\cos kt}{k} + \frac{1}{k}\int \cos kt\, dt =$$

$$= \left[\frac{-t \cdot \cos kt}{k} + \underbrace{\frac{1}{k^2} \cdot \sin kt}_{0}\right]_{\pi}^{2\pi} = \boxed{\frac{(-t) - (-1)^k \cdot (-t)}{k}}$$

$$\Rightarrow \underbrace{\left[\frac{\sin kt}{k}\right]_0^{\pi}}_{0} + \left[\frac{2 \cdot (-\cos kt)}{k}\right]_{\pi}^{2\pi} + \frac{(-t) + (+1)^k \cdot (+t)}{k} =$$

$$= \frac{2 \cdot (-1 + 1)}{k} \quad ?$$

$$f(t) = \frac{3\pi}{4} + \sum_{k=1}^{\infty} \frac{(-1)^k - 1}{\pi k^2} \cos kt + \frac{1}{k} \cdot \sin kt\Big) \, ; \, t \in R$$