str. 151 př. 1

a)

$$\int_2^{\infty} \frac{1}{x^3}\,dx = \left[-\frac{1}{2x^2}\right]_0^2$$

$$\lim_{x\to\infty}\left(-\frac{1}{2x^2}\right) - \lim_{x\to 0}\left(-\frac{1}{2x^2}\right) = 0 - \left(-\frac{1}{8}\right) = \frac{1}{8}$$

b)

$$\int_{-\infty}^{0} \frac{dx}{(x-1)^4} = \left[-\frac{1}{3(x-1)^3}\right]_{-\infty}^{0}$$

$$\lim_{x\to 0}\left(-\frac{1}{3(x-1)^3}\right) - \lim_{x\to-\infty}\left(-\frac{1}{3(x-1)^3}\right) = \frac{1}{3} - 0 = \frac{1}{3}$$

c)

$$\int_{-8}^{0} \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2}\sqrt[3]{x^2}\right]_{-8}^{0}$$

$$\lim_{x\to 0}\left(\frac{3}{2}\sqrt[3]{x^2}\right) - \lim_{x\to -8}\left(\frac{3}{2}\sqrt[3]{x^2}\right) = 0 - 6 = -6$$

d)

$$\int_0^{\infty} \frac{1}{\sqrt[3]{x^4}}\,dx = \left[4x^{\frac{1}{4}}\right]_0^{\infty}$$

$$\lim_{x\to\infty} 4x^{\frac{1}{4}} - \lim_{x\to 0} 4x^{\frac{1}{4}} = \infty - 0 = \infty$$

e)

$$\int_0^{\infty} \sin(x)\,dx = \left[-\cos(x)\right]_0^{\infty}$$

$$\lim_{x\to\infty}\left(-\cos(x)\right) - \lim_{x\to 0}\left(-\cos(x)\right) = neexistuje + 1 \Rightarrow neexistuje$$

f)

$$\int_{-\infty}^{0} e^x\,dx = \left[e^x\right]_{-\infty}^{0}$$

$$\lim_{x\to 0} e^x - \lim_{x\to-\infty} e^x = 1 - (-0) = 1$$

g)

$$\int_{-\infty}^{\infty} \frac{x+1}{x^2-4x+5}\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{2x-4}{x^2-4x+5}\,dx + \int_{-\infty}^{\infty} \frac{3}{x^2-4x+5}\,dx$$

$$\frac{1}{2}\int_{-\infty}^{\infty} \frac{2x-4}{x^2-4x+5}\,dx = \frac{1}{2}\ln\left|x^2-4x+5\right|$$

$$\int_{-\infty}^{\infty} \frac{3}{x^2-4x+5}\,dx = 3\int_{-\infty}^{\infty} \frac{1}{(x-2)^2+1}\,dx = 3\arctan(x-2)$$

$$\int_{-\infty}^{\infty} \frac{x+1}{x^2-4x+5}\,dx = \left[\frac{1}{2}\ln\left|x^2-4x+5\right| + 3\arctan(x-2)\right]_{-\infty}^{\infty}$$

$$\lim_{x\to\infty}\left(\frac{1}{2}\ln\left|x^2-4x+5\right| + 3\arctan(x-2)\right) - \lim_{x\to-\infty}\left(\frac{1}{2}\ln\left|x^2-4x+5\right| + 3\arctan(x-2)\right) =$$

$$= \left(\infty + \frac{3}{2}\pi\right) - \left(\infty - \frac{3}{2}\pi\right) = \infty - \infty + 3\pi \Rightarrow neexistuje$$

str. 151 př. 2 (per-partes, l'Hospital)

a)

$$\int_{1/2}^{\infty} x.e^{-2x} = e^{-2x}\left(-\frac{1}{2}x-\frac{1}{4}\right)=\left[-\frac{2x+1}{4e^{2x}}\right]_{1/2}^{\infty}$$

$$\lim_{x\to\infty}\left(-\frac{2x+1}{4e^{2x}}\right)-\lim_{x\to 1/2}\left(-\frac{2x+1}{4e^{2x}}\right)=\lim_{x\to\infty}\left(-\frac{2}{8e^{2x}}\right)-\lim_{x\to 1/2}\left(-\frac{2x+1}{4e^{2x}}\right)=\left(-\frac{1}{\infty}\right)-\left(-\frac{2}{4e^{1}}\right)=0+\frac{1}{2e}=\frac{1}{2e}$$

b)

$$\int_{1}^{\infty}\frac{\ln x}{x^2}dx=\int_{1}^{\infty}x^{-2}\ln x=\left[-\frac{1}{x}\ln(x)-\frac{1}{x}\right]_{1}^{\infty}$$

$$\lim_{x\to\infty}\left(-\frac{1}{x}\ln(x)-\frac{1}{x}\right)-\lim_{x\to 1}\left(-\frac{1}{x}\ln(x)-\frac{1}{x}\right)=(0)-(-1)=1$$

c)

$$\int_{0}^{\infty}x^4e^{-x}dx=\left[\left(-e^{-x}\right)\left(x^4+4x^3+12x^2+24x+24\right)\right]_{0}^{\infty}$$

$$\lim_{x\to\infty}\left(-e^{-x}\right)\left(x^4+4x^3+12x^2+24x+24\right)-\lim_{x\to 0}\left(-e^{-x}\right)\left(x^4+4x^3+12x^2+24x+24\right)=0-(-24)=24$$

d)

$$\int_{1}^{\infty}x^2\ln^2 x dx=\left[-\left(\frac{\ln^2 x}{x}+\frac{2\ln x}{x}+\frac{2}{x}\right)\right]_{1}^{\infty}$$

$$\lim_{x\to\infty}-\left(\frac{\ln^2 x}{x}+\frac{2\ln x}{x}+\frac{2}{x}\right)+\lim_{x\to 1}\left(\frac{\ln^2 x}{x}+\frac{2\ln x}{x}+\frac{2}{x}\right)=0+2=2$$

e)

$$\int_{0}^{\infty}e^{-2x}\cos(3x)dx=\left[\frac{3}{13}e^{-2x}\sin x-\frac{2}{13}e^{-2x}\cos x\right]_{0}^{\infty}$$

$$\lim_{x\to\infty}\left(\frac{3}{13}e^{-2x}\sin x-\frac{2}{13}e^{-2x}\cos x\right)-\lim_{x\to 0}\left(\frac{3}{13}e^{-2x}\sin x-\frac{2}{13}e^{-2x}\cos x\right)=0-\left(-\frac{2}{13}\right)=\frac{2}{13}$$

f)

$$\int_{0}^{\infty}e^{-2x}\sin(3x)dx=\left[-\frac{3}{13}e^{-2x}\cos x-\frac{2}{13}e^{-2x}\sin x\right]_{0}^{\infty}$$

$$\lim_{x\to\infty}\left(-\frac{3}{13}e^{-2x}\cos x-\frac{2}{13}e^{-2x}\sin x\right)-\lim_{x\to 0}\left(-\frac{3}{13}e^{-2x}\cos x-\frac{2}{13}e^{-2x}\sin x\right)=0-\left(-\frac{3}{13}\right)=\frac{3}{13}$$

str. 151 př. 3

a)

$$\int_{-\infty}^{\infty}\frac{1}{x^2-2x+3}dx=\int_{-\infty}^{\infty}\frac{1}{2\left(\left(\frac{x-1}{\sqrt{2}}\right)^2+1\right)}dx=\left[\frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{4}(2x-2)\right)\right]_{-\infty}^{\infty}\ ?$$

$$\lim_{x\to\infty}\left(\frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{4}(2x-2)\right)\right)-\lim_{x\to-\infty}\left(\frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{4}(2x-2)\right)\right)=\frac{\sqrt{2}}{2}\pi$$

b)

$$\int_{-\infty}^{\infty} \frac{2x+5}{x^2+2x+5} = \int_{-\infty}^{\infty} \frac{2x+2}{x^2+2x+5}\,dx = \int_{-\infty}^{\infty} \frac{3}{x^2+2+x}\,dx.$$

$$\int_{-\infty}^{\infty} \frac{2x+2}{x^2+2x+5}\,dx = \ln\left|x^2+2x+5\right|$$

$$\int_{-\infty}^{\infty} \frac{3}{x^2+2x+5}\,dx = \frac{3}{4}\int_{-\infty}^{\infty} \frac{1}{\left(\frac{x+2}{2}\right)^2+1}dx = \frac{3}{4}\arctan\left(\frac{x+2}{2}\right)$$

$$\int_{-\infty}^{\infty} \frac{2x+5}{x^2+2x+5}dx = \left[\ln\left|x^2+2x+5\right| + \frac{3}{4}\arctan\left(\frac{x+2}{2}\right)\right]_{-\infty}^{\infty}$$

$$\lim_{x\to\infty}\left(\ln\left|x^2+2x+5\right| + \frac{3}{4}\arctan\left(\frac{x+2}{2}\right)\right) - \lim_{x\to-\infty}\left(\ln\left|x^2+2x+5\right| + \frac{3}{4}\arctan\left(\frac{x+2}{2}\right)\right) =$$

$$= \left(\infty + \frac{3}{4}\frac{\pi}{4}\right) - \left(8 + \frac{3}{4}\frac{\pi}{4}\right) = \infty$$

c)

$$\int_{-\infty}^{-2} \frac{dx}{x^2-2x-3} = \int_{-8}^{2} -\frac{1}{4(x+1)} + \frac{1}{4(x-3)} = \left[\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|\right]_{-\infty}^{-2}$$

$$\lim_{x\to-2}\left(\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|\right) - \lim_{x\to-\infty}\left(\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|\right) = \left(\frac{1}{4}\ln 5\right) - 0 = \frac{1}{4}\ln 5$$

d)

$$\int_{0}^{\infty} \frac{x+4}{(x+1)(x^2+3x+2)}dx = \int_{0}^{\infty} \frac{3}{(x+1)^2} - \frac{2}{(x+1)} + \frac{2}{(x+2)}dx = \left[-\frac{3}{(x+1)} + 2\ln\frac{|x+2|}{|x+1|}\right]_{0}^{\infty}$$

$$\lim_{x\to\infty}\left(-\frac{3}{(x+1)} + 2\ln\frac{|x+2|}{|x+1|}\right) - \lim_{x\to0}\left(-\frac{3}{(x+1)} + 2\ln\frac{|x+2|}{|x+1|}\right) = (0) - (-3 + 2\ln 2) = 3 - \ln 4$$

str. 151 př. 4 (substituce)

a)

$$\int_{0}^{\infty} \frac{dx}{e^{2x}+4e^x+3} = \begin{vmatrix} e^x = t \\ dx = dt \end{vmatrix} = \int_{0}^{\infty} \frac{1}{2(t+1)} - \frac{1}{2(t+3)}\,dt = ?\,chyba$$

b)

$$\int_{0}^{\infty} \frac{dx}{e^x+e^{-x}} = \begin{vmatrix} t = e^x \\ dx = \frac{1}{t}dt \end{vmatrix} = \int_{0}^{\infty} \frac{1}{t+\frac{1}{t}}.\frac{dt}{t} = \int_{0}^{\infty} \frac{1}{t^2+1} = \left[\arctan\left(e^x\right)\right]_{0}^{\infty}$$

$$\lim_{x\to\infty}\arctan\left(e^x\right) - \lim_{x\to0}\arctan\left(e^x\right) = \left(\frac{\pi}{2}\right) - \left(\frac{\pi}{4}\right) = \frac{\pi}{4}$$

c)

$$\int_0^e \frac{dx}{x\left(\ln^2 x+2\ln x+5\right)}=\left|\begin{array}{l} t=\ln x \\ \frac{1}{x}dx=dt \end{array}\right|=\int_0^e \frac{1}{\left(\frac{t+1}{2}\right)^2+1}dt=\left[\frac{1}{2}\arctan\left(\frac{\ln x+1}{2}\right)\right]_0^e$$

$$\lim_{x\to e}\frac{1}{2}\arctan\left(\frac{\ln x+1}{2}\right)-\lim_{x\to 0}\frac{1}{2}\arctan\left(\frac{-\infty+1}{2}\right)=\left(\frac{\pi}{8}\right)-\left(-\frac{\pi}{4}\right)=\frac{3}{8}\pi$$

$$\lim_{x\to 0}\ln x=-\infty$$

d)

$$\int_1^\infty \frac{dx}{x\left(\ln^2 x+3\ln x+2\right)}=\left|\begin{array}{l} \ln x=t \\ \frac{1}{x}dx=dt \end{array}\right|=\int_1^\infty \frac{dt}{t^2+3t+2}=\int_1^8 \frac{1}{t+1}-\frac{1}{t+2}dt=\left[\ln\left|\frac{\ln x+1}{\ln x+2}\right|\right]_1^8$$

$$\lim_{x\to\infty}\left(\ln\left|\frac{\ln x+1}{\ln x+2}\right|\right)-\lim_{x\to 1}\left(\ln\left|\frac{\ln x+1}{\ln x+2}\right|\right)=0-\left(\ln\frac{1}{2}\right)=\ln 2$$

e)

$$\lim_{x\to\infty}\left(\ln\left|\frac{\ln x+1}{\ln x+2}\right|\right)-\lim_{x\to 1}\left(\ln\left|\frac{\ln x+1}{\ln x+2}\right|\right)=0-\left(\ln\frac{1}{2}\right)=\ln 2$$

$$\int_3^\infty \frac{dx}{(x+1)\sqrt{x}}=\left|\begin{array}{l} x=t^2 \\ dx=2tdt \end{array}\right|=\int_3^\infty \frac{2t}{\left(t^2+1\right)t}dt=2\arctan(t)=\left[2\arctan\left(\sqrt{x}\right)\right]_3^\infty$$

$$\lim_{x\to\infty}2\arctan\left(\sqrt{x}\right)-\lim_{x\to 3}2\arctan\left(\sqrt{x}\right)=(\pi)-\left(\frac{2}{3}\pi\right)=\frac{1}{3}\pi$$

f)

$$\int_6^\infty \frac{dx}{(x-1)\sqrt{x+3}}=\left|\begin{array}{l} x+3=t^2 \\ x=t^2-3 \\ dx=2tdt \end{array}\right|=\int_6^\infty \frac{2t}{\left(t^2-3-1\right)t}dt=2\int_6^\infty \frac{1}{(t-2)(t+2)}=2\int_6^\infty \frac{1}{4(t-2)}-\frac{1}{4(t+2)}dt$$

$$=\frac{1}{2}\ln|t-2|-\frac{1}{2}|\ln t+2|=\left[\frac{1}{2}\ln\left|\frac{t-2}{t+2}\right|\right]_0^\infty$$