str. 108 př. 1

$$\int \left(x^3 - 2x^2 + 5x - 3x\right) dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + C, x \in R$$

$$\int \frac{x^3 - x + 4}{x^2} dx = \int \left(x - x^{-1} + 4x^{-2}\right) dx = \frac{1}{2}x^2 - \ln|x| - \frac{4}{x} + C, x \in R - \{0\}$$

$$\int \frac{2x-5}{x^5} dx = \int \left(2x^{-4} - 5x^{-5}\right) dx = -\frac{2}{3x^3} + \frac{5}{4x^4} + C, x \in R - \{0\}$$

$$\int \sqrt[4]{x} dx = \int x^{\frac{1}{4}} dx = \frac{4}{5}x^{\frac{5}{4}} + C = \frac{4}{5}x.\sqrt[4]{x} + C, x > 0$$

$$\int \sqrt[5]{x^3} dx = \int x^{\frac{3}{5}} dx = \frac{5}{8}x^{\frac{8}{5}} + C = \frac{5}{8}x.\sqrt[5]{x^3} + C, x \in R$$

$$\int \frac{dx}{\sqrt[3]{x}} = \int x^{-\frac{1}{3}} dx = \frac{3}{2}x^{\frac{2}{3}} + C = \frac{3}{2}\sqrt[3]{x^2} + C, x \in R - \{0\}$$

$$\int \left(3e^x + 4\sin(x) - 2\cos(x)\right) dx = 3e^x - 4\cos(x) - 2\sin(x) + C, x \in R$$

$$\int \frac{2x^2+5}{x^2+1} dx = \int \left(2 + \frac{3}{x^2+1}\right) dx = 2x + 3\arctan(x) + C, x \in R$$

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$$\int (3x-4)^6 dx = \begin{vmatrix} 3x-4=t \\ 3dx = dt \\ dx = dt/3 \end{vmatrix} = \frac{1}{3}\int t^6 dt = \frac{1}{3}\frac{t^7}{7} + C = \frac{(3x-4)^7}{21} + C, x \in R$$

$$\int \frac{dx}{(3x-1)} = \int (3x-1)^{-1} dx = \frac{\ln|3x-1|}{3} + C, x \in R - \left\{\frac{1}{3}\right\}$$

$$\int \frac{dx}{(2x+1)^5} = \int (2x+1)^{-5} dx = -\frac{1}{4.2}(2x+1)^{-4} + C = \frac{1}{8}\ln^4|2x+1| + C, x \in \left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$$

$$\int \sqrt[4]{2-x} dx = \int (2-x)^{\frac{1}{4}} dx = -\frac{4}{5}(2-x)^{\frac{5}{4}} + C, x \in (-\infty, 2)$$

$$\int \left(2\cos(3x) - 6\sin(2x)\right) dx = 2\int \cos(3x) dx - 6\int \sin(2x) dx = \frac{2}{3}\sin(3x) + 3\cos(2x) + C, x \in R$$

$$\int \left(3\sin\left(\frac{x}{3}\right) - \cos\left(\frac{x}{2}\right)\right) dx = 3\int \sin\left(\frac{x}{3}\right) dx - \int \cos\left(\frac{x}{2}\right) dx = -9\cos\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right) + C, x \in R$$

$$\int \left(5e^{2x} + 3e^{-x}\right) dx = 5\int e^{2x} dx + 3\int e^{-x} dx = \frac{5}{2}e^{2x} - 3e^{-x} + C, x \in R$$

$$\int \left(3^x + 3.2^{-2x}\right) dx = \int 3^x dx + 3\int 2^{-2x} = \frac{1}{\ln 3}.3^x - \frac{3.2^{-2x}}{2\ln 2} + C, x \in R$$

$$\int \sinh(ax) dx = \frac{1}{a}\cosh(ax) + C, x \in R$$

$$\int \cosh(ax) dx = \frac{1}{a}\sinh(ax) + C, x \in R$$

str. 108 př. 3

$$\sin^2(x) = \frac{1}{2}\left(1-\cos(2x)\right)$$

$$\cos^2(x) = \frac{1}{2}\left(1+\cos(2x)\right)$$

$$\int \sin^2(x) = \frac{1}{2}\int 1-\cos(2x)\,dx = \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C, x \in R$$

$$\int \cos^2(x) = \frac{1}{2}\int 1+\cos(2x)\,dx = \frac{1}{2}\left(x + \frac{\sin(2x)}{2}\right) + C, x \in R$$

str. 108 př. 4

$$\int (x-2)\sin(2x)\,dx = -\frac{1}{2}(x-2)\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx =$$
$$= -\frac{1}{2}(x-2)\cos(2x) + \frac{1}{4}\sin(2x) + C, x \in R$$

$u = (x-2) \quad u' = 1$
$v' = \sin(2x) \quad v = -\frac{1}{2}\cos(2x)$

$$\int (x-2)\sin(2x)\,dx = -\frac{1}{2}(x-2)\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx =$$
$$= -\frac{1}{2}(x-2)\cos(2x) + \frac{1}{4}\sin(2x) + C, x \in R$$

$u = (x+1) \quad x' = 1$
$v' = \cos\left(\frac{x}{3}\right) \quad v = 3\sin\left(\frac{x}{3}\right)$

$$\int (3x-1)e^{3x}\,dx = \frac{1}{3}(3x-1)e^{3x} - \frac{3}{3}\int e^{3x}\,dx =$$
$$= \frac{1}{3}(3x-1)e^{3x} - \frac{1}{3}e^{3x} + C = \frac{1}{3}(3x-2)e^{3x} + C, x \in R$$

$u = 3x-1 \quad u' = 3$
$v' = e^{3x} \quad v = \frac{e^{3x}}{3}$

$$\int \left(x+\sqrt{x}\right).\ln(x)\,dx = \ln x\left(\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}\right) - \int \frac{1}{x}\left(\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}\right)dx =$$
$$= \ln x\left(\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}\right) - \int\left(\frac{x^2}{2} + \frac{2}{3}x^{\frac{1}{2}}\right)dx = \ln x\left(\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}\right) - \frac{x^2}{4} - \frac{4}{9}x^{\frac{3}{2}} + C =$$
$$= \left(\frac{1}{2}x^2 + \frac{2}{3}\sqrt{x^3}\right).\ln(x) - \frac{1}{4}x^2 - \frac{4}{9}\sqrt{x^3} + C, x \in (0, +\infty)$$

$u = \ln(x) \quad u' = \frac{1}{x}$
$v' = \left(x+\sqrt{x}\right) \quad v = \frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}$

$$\int \frac{\ln(x)}{x^2}\,dx = -\frac{\ln(x)}{x} + \int\left(\frac{1}{x^2}\right)dx = -\frac{\ln(x)}{x} - \frac{1}{x} = -\frac{1}{x}\left(\ln(x)+1\right), x \in (0, +\infty)$$

$u = \ln(x) \quad u' = \frac{1}{x}$
$v' = x^{-2} \quad v = -\frac{1}{x}$

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$$\int (x^2-x)\sin\left(\frac{x}{2}\right)dx = -2(x^2-x)\cos\left(\frac{x}{2}\right)+2\int(2\text{x-}1)\cos\left(\frac{x}{2}\right)dx =$$

$$= -2(x^2-x)\cos\left(\frac{x}{2}\right)+2\left((2x-1)2\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)=$$

$$= -2(x^2-x)\cos\left(\frac{x}{2}\right)+2\left((2x-1)2\sin\left(\frac{x}{2}\right)+8.2\cos\left(\frac{x}{2}\right)\right)+C=$$

$$= -2\cos\left(\frac{x}{2}\right)(x^2-x-9)+4(2x-1)\sin\left(\frac{x}{2}\right)+C, x\in R$$

$u = x^2 - x \quad u' = 2x-1$

$v' = \sin\left(\frac{x}{2}\right) \quad v = -2\cos\left(\frac{x}{2}\right)$

$u = 2x-1 \quad u' = 2$

$v' = \cos\left(\frac{x}{2}\right) \quad v = 2\sin\left(\frac{x}{2}\right)$

$$\int(2x^2+3)\cos(2x)dx = \frac{1}{2}(2x^2+3)\sin(2x)-\frac{4}{2}\int x.\sin(2x)dx =$$

$$= \frac{2x^2+3}{2}\sin(2x)-2\left(-\frac{1}{2}x\cos(2x)+\frac{1}{2}\int\cos(2x)dx\right)=$$

$$= \frac{2x^2+3}{2}\sin(2x)+1\left(x.\cos(2x)-\frac{1}{2}\sin(2x)\right)+C=$$

$$= (x^2+1)\sin(2x)+x.\cos(2x)+C, x\in R$$

$u = 2x^2+3 \quad u' = 4x$

$v' = \cos(2x) \quad v = \frac{\sin(2x)}{2}$

$u = x \quad u' = 1$

$v' = \sin(2x) \quad v = -\frac{1}{2}\cos(2x)$

$$\int(x^2+x+1)e^{-x}dx = -\frac{(x^2+x+1)}{e^x}-\int(2x+1)e^{-x}dx =$$

$$= -\frac{x^2+x+1}{e^x}-\left(-\frac{2x+1}{e^x}-2\int e^{-x}dx\right) = -\frac{x^2+x+1+(2x+1)}{e^x}-2e^{-x}+C=$$

$$-\frac{x^2+3x+4}{e^x}+C, x\in R$$

$u = (x^2+x+1) \quad u' = 2x+1$

$v' = e^{-x} \quad v = -e^{-x}$

$u = 2x+1 \quad u' = 2$

$v' = e^{-x} \quad v = -e^{-x}$

$$\int\ln^2(x)dx = x.\ln^2(x)-2\int x.\frac{\ln(x)}{x}dx = x.\ln^2(x)-2\left(x.\ln(x)-\int\frac{x}{x}dx\right)=$$

$$x.\ln^2(x)-2x.\ln(x)+2x = x(\ln^2 x-2\ln x+2)+C, x\in(0,\infty)$$

$u = \ln^2 x \quad u' = 2\frac{\ln(x)}{x}$

$v' = 1 \quad v = x$

$u = \ln x \quad u' = \frac{1}{x}$

$v' = 1 \quad v = x$

$$\int x.\ln^3(x)dx = \frac{x^2}{2}.\ln^3(x) - \frac{3}{2}\int x^2.\frac{\ln^2(x)}{x}dx =$$

$$u = \ln^3 x \quad u' = \frac{3\ln^2 x}{x} \qquad v' = x \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2}\ln^3(x) - \frac{3}{2}\left(\frac{x^2}{2}\ln^2(x) - \int \frac{x^2}{2}.\frac{2\ln x}{x}dx\right) =$$

$$u = \ln^3 x \quad u' = \frac{3\ln^2 x}{x} \qquad v' = x \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2}\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{2}\left(\frac{x^2}{2}\ln(x) - \frac{1}{2}\int x^2.\frac{1}{x}dx\right) =$$

$$u = \ln^3 x \quad u' = \frac{3\ln^2 x}{x} \qquad v' = x \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2}\ln^3(x) - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln(x) - \frac{3}{4}\frac{x^2}{2} + C =$$

$$= \frac{1}{2}x^2\left(\ln^3(x) - \frac{3}{2}\ln^2(x) + \frac{3}{2}\ln(x) - \frac{3}{4}\right) + C, x \in (0, +\infty)$$

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$$\int e^{2x}\cos\left(\frac{x}{3}\right)dx = 3e^{2x}\sin\left(\frac{x}{3}\right) - 6\int e^{2x}\sin\left(\frac{x}{3}\right)dx =$$

$$u = e^{2x} \quad u' = 2e^{2x} \qquad v' = \cos\left(\frac{x}{3}\right) \quad v = 3\sin\left(\frac{x}{3}\right)$$

$$= 3e^{2x}\sin\left(\frac{x}{3}\right) - 6\left(-3e^{2x}\cos\left(\frac{x}{3}\right) + 6\int e^{2x}\cos\left(\frac{x}{3}\right)dx\right) =$$

$$= 3e^{2x}\sin\left(\frac{x}{3}\right) + 18e^{2x}\cos\left(\frac{x}{3}\right) - 36\int e^{2x}\cos\left(\frac{x}{3}\right)dx$$

$$u = e^{2x} \quad u' = 2e^{2x} \qquad v' = \sin\left(\frac{x}{3}\right) \quad v = -3\cos\left(\frac{x}{3x}\right)$$

$$\int e^{2x}\cos\left(\frac{x}{3}\right)dx = 3e^{2x}\sin\left(\frac{x}{3}\right) + 18e^{2x}\cos\left(\frac{x}{3}\right) - 36\int e^{2x}\cos\left(\frac{x}{3}\right)dx$$

$$37\int e^{2x}\cos\left(\frac{x}{3}\right)dx = 3e^{2x}\sin\left(\frac{x}{3}\right) + 18e^{2x}\cos\left(\frac{x}{3}\right)$$

$$\int e^{2x}\cos\left(\frac{x}{3}\right)dx = \frac{3}{37}e^{2x}\left(\sin\left(\frac{x}{3}\right) + 6\cos\left(\frac{x}{3}\right)\right), x \in R$$

$$\int e^{-x}\sin(2x)dx = -e^{-x}\sin(2x) + 2\int e^{-x}\cos(2x)dx =$$

$$u' = e^{-x} \quad u = -e^{-x} \qquad v = \sin(2x) \quad v' = 2\cos(2x)$$

$$= -e^{-x}\sin(2x) + 2\left(-e^{-x}.\cos(2x)\right) - 4\int e^{-x}\sin(2x)dx =$$

$$= -e^{-x}\sin(2x) + 2e^{-x}\cos(2x) + C$$

$$u' = e^{-x} \quad u = -e^{-x} \qquad v = \cos(2x) \quad v' = -2\sin(2x)$$

$$5\int e^{-x}\sin(2x)dx = -e^{-x}\sin(2x) + 2e^{-x}\cos(2x) + C$$

$$\int e^{-x}\sin(2x)dx = -\frac{1}{5}e^{-x}\left(\sin(2x) + 2\cos(2x)\right) + C, x \in R$$

$$\int \frac{\ln(x)}{x}dx = \ln|x|.\ln x - \int \frac{1}{x}.\ln|x|dx$$

$$u' = \frac{1}{x} \quad u = \ln|x| \qquad v = \ln x \quad v' = \frac{1}{x}$$

$$2\int \frac{\ln(x)}{x}dx = \ln|x|.\ln x + C$$

$$\int \frac{\ln(x)}{x}dx = \frac{\ln^2 x}{2} + C, x \in (0, +\infty)$$

str. 108 př. 7

$$\int \frac{x+1}{x^2+2x+4}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+4} = dx = \frac{1}{2}\ln\left(x^2+2x+4\right)+C, x \in R$$

$$\int \frac{2x^2}{x^3-1}dx = \frac{2}{3}\ln\left|x^3-1\right|+C, x \in R-\{1\}$$

$$\int \cot g\left(2x\right)dx = \int \frac{\cos\left(2x\right)}{\sin\left(2x\right)}dx = \frac{1}{2}\ln\left|\sin 2x\right|+C, x \in \left(0+\frac{k\pi}{2}, \frac{\pi}{2}+\frac{k\pi}{2}\right)$$

str. 108 př. 8

$$\int 2x\left(x^2-1\right)^4 dx = \left|\begin{matrix}\left(1-x^2\right)=t \\ 2x^2dx=dt \\ x^2dx=dt/\left(-2\right)\end{matrix}\right| = \frac{1}{2}2\int t^4dt = \frac{1}{5}t^5+C = \frac{\left(x^2-1\right)^5}{5}+C, x \in R$$

$$\int x.e^{x^2+1}dx = \left|\begin{matrix}x^2+1=t \\ 2xdx=dt \\ xdx=dt/2\end{matrix}\right| = \frac{1}{2}2\int t^4dt = \frac{1}{5}t^5dt = \frac{1}{5}\left(x^2-1\right)+C, x \in R$$

$$\int x\sqrt{1-x^2}dx = \left|\begin{matrix}1+x^3=t \\ -2x^2dx=dt \\ x^2dx=dt/\left(-2\right)\end{matrix}\right| = -\frac{1}{2}\int t^{\frac{1}{2}}dt = -\frac{1}{2}.\frac{2}{3}t^{\frac{3}{2}}+C = -\frac{1}{3}\sqrt[3]{\left(1-x^2\right)^2}+C, x \in \left(-1,1\right)$$

$$6\int x^2\sqrt{1+x^3}dx = \left|\begin{matrix}1+x^3=t \\ 3x^4dx=dt \\ x^4dx=dt/3\end{matrix}\right| = 2\int t^{\frac{1}{2}}dt = 2.\frac{2}{3}t^{\frac{3}{2}}+C = \frac{4}{3}\sqrt{\left(1+x^3\right)^3}+C, x \in R$$

$$\int \frac{x^2}{\sqrt{x^3+8}}dx = \left|\begin{matrix}x^3+8=t \\ 3x^5dx=dt \\ x^5dx=dt/3\end{matrix}\right| = \frac{1}{3}\int t^{-\frac{1}{2}}dt = \frac{1}{3}2t^{\frac{1}{2}}+C = \frac{2}{3}\sqrt{x^3+8}+C, x \in \left(-2,+\infty\right)$$

$$\int \frac{4x+4}{\sqrt[3]{x^2+2x+2}}dx = 4\int x+1.\left(x^2+2x+2\right)^{-\frac{1}{3}} = \left|\begin{matrix}x^2+2x+2=t \\ 2xdx=dt \\ xdx=dt/2\end{matrix}\right| = \frac{4}{2}\int t^{-\frac{1}{3}}dt =$$

$$= 2\frac{3}{2}t^{\frac{2}{3}}+C = 3\sqrt[3]{x^2+2x+2}+C, x \in R$$

$$\int \text{in}^7\left(x\right)\cos\left(x\right)dx = \left|\begin{matrix}\sin\left(x\right)=t \\ \cos\left(x\right)dx=dt\end{matrix}\right| = \int t^7dt = \frac{1}{8}t^8+C = \frac{\sin^8\left(x\right)}{8}+C, x \in R$$

$$\int \sin\left(x\right).\cos^4\left(x\right)dx = \left|\begin{matrix}\cos\left(x\right)=t \\ -\sin\left(x\right)dx=dt\end{matrix}\right| = -\int t^4dt = -\frac{1}{5}t^5+C = -\frac{\cos^5\left(x\right)}{5}+C, x \in R$$

$$\int \frac{\ln^3\left(x\right)}{x}dx = \left|\begin{matrix}\ln x=t \\ dx=dt\end{matrix}\right| = \int t^3dt = \frac{1}{4}t^4+C = \frac{\ln^4\left(x\right)}{4}+C, x \in \left(0,+\infty\right)$$

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$$\int \operatorname{arc}\cot g(x)\,dx = x.arc\cot g(x) + \int \left(1+x^2\right)^{-1} dx = \begin{vmatrix} 1+x^2 = t \\ 2xdx = dt \\ xdx = dt/2 \end{vmatrix} =$$

$$= x.arc\cot g(x) + \frac{1}{2}\ln\left(x^2+1\right) + C, x \in R$$

$$\int \arcsin(x)\,dx = x.\arcsin(x) - \int x.\frac{1}{\sqrt{1-x^2}}\,dx = \begin{vmatrix} 1-x^2 = t \\ -2xdx = dt \\ xdx = dt/(-2) \end{vmatrix} =$$

$$= x.\arcsin(x) + \frac{1}{2}2t^{\frac{1}{2}} = x.\arcsin(x) + \sqrt{1-x^2} + C, x \in (-1,1)$$

$$\int \arccos(x)\,dx = x.\arccos(x) + \int \left(1-x^2\right)^{-\frac{1}{2}} dx = \begin{vmatrix} 1-x^2 = t \\ -2xdx = dt \\ xdx = dt/(-2) \end{vmatrix} =$$

$$= x.\arccos(x) - \frac{1}{2}\int t^{-\frac{1}{2}} dt = x.\arccos(x) - \frac{1}{2}2t^{\frac{1}{2}} + C =$$

$$= x.\arccos(x) - \sqrt{1-x^2} + C, x \in (-1,1)$$