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$$\lim_{x\to 0}=\frac{\sin(3x)}{\sin(4x)}=\lim_{x\to 0}\frac{3\cos(3x)}{4\cos(4x)}=\frac{3.1}{4.1}=\frac{3}{4}$$

$$\lim_{x\to 1}\frac{\sin(\pi x)}{\ln(x)}=\lim_{x\to 1}\frac{\pi.\cos(\pi x)}{\frac{1}{x}}=\frac{\pi}{1}=\pi$$

$$\lim_{x\to o}\frac{e^x-e^{-x}}{\sin(x)}=\lim_{x\to 1}\frac{e^x+e^x}{\cos(x)}=\frac{1+1}{1}=2$$

$$\lim_{x\to 1}\frac{\ln^2(x)}{x-1}=\lim_{x\to 1}\frac{2.\ln(x)\frac{1}{x}}{1}=\frac{2.0.1}{1}=0$$

$$\lim_{x\to 1}\frac{\ln(x)}{(x-1)^2}=\lim_{x\to 1}\frac{\frac{1}{x}}{2x-2}=\frac{1}{2x(x-1)}=neexistuje$$

$$\lim_{x\to 0}\frac{\sinh(2x)}{3x-2}=\lim_{x\to 0}\frac{2\cosh(2x)}{3}=\frac{2.1}{3}=\frac{2}{3}$$

$$\lim_{x\to\infty}\frac{\ln(x)}{x+2}=\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=\frac{1}{\infty}=0$$

$$\lim_{x\to\infty}\frac{e^{2x}}{5x+3}=\lim_{x\to\infty}\frac{2e^{2x}}{5}=\infty$$

$$\lim_{x\to\infty}\frac{e^x}{e^{2x}+1}=\lim_{x\to\infty}\frac{e^x}{2e^{2x}}=\frac{1}{2e^x}=\frac{1}{\infty}=0$$

$$\lim_{x\to 0}\frac{x-arctgx}{x^3}=\lim_{x\to 0}\frac{1-\frac{1}{1+x^2}}{3x^2}=\lim_{x\to 0}\frac{\frac{1}{\frac{1}{x^2}+1}}{3.1}=\frac{\frac{1}{0+1}}{3}=\frac{1}{3}$$

$$\lim_{x\to\infty}\frac{\ln(x)}{\sqrt{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}=\lim_{x\to\infty}\frac{1}{x}.\frac{2\sqrt{x}}{1}=\frac{2\sqrt{x}}{x}=\frac{2}{\sqrt{x}}=0$$

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$$\lim_{x\to 0}\frac{1-\cos(x)}{x^2}=\lim_{x\to 0}\frac{\sin(x)}{2x}=\lim_{x\to 0}\frac{\cos(x)}{2}=\frac{1}{2}$$

$$\lim_{x\to\infty}\frac{x^3-2x^2}{e^x}=\lim_{x\to\infty}\frac{3x^2-4x}{e^x}=\lim_{x\to\infty}\frac{6x-4}{e^x}=\lim_{x\to\infty}\frac{6}{e^x}=\frac{6}{\infty}=0$$

$$\lim_{x\to 0}\frac{\cosh(x)-\cos(x)}{x^2}=\lim_{x\to 0}\frac{\sinh(x)+\sin(x)}{2x}=\lim_{x\to 0}\frac{\cosh(x)+\cos(x)}{2}=\frac{1+1}{2}=1$$

$$\lim_{x\to 3}\frac{\ln(x^2-8)}{x^2-3x}=\lim_{x\to 3}\frac{2x.\frac{1}{x^2-8}}{2x-3}=\lim_{x\to 3}\frac{6.\frac{1}{9-8}}{6-3}=\frac{6}{3}=2$$

$$\lim_{x\to\infty}\frac{\ln(x^2-8)}{x^2-3x}=\lim_{x\to\infty}\frac{\frac{2x}{x^2-8}}{2x-3}=\lim_{x\to\infty}\frac{2x}{2x^3-3x^2-16x+24}=\lim_{x\to\infty}\frac{2}{6x^2-6-16}=\frac{2}{\infty}=0$$

$$\lim_{x\to\infty}\frac{x-arctg(x)}{x^3}=\lim_{x\to\infty}\frac{1-\frac{1}{1+x^2}}{3x^2}=\lim_{x\to\infty}\frac{\frac{1}{x^2}-\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{x^2}{x^2}}}{\frac{3x^2}{x^2}}=\frac{0-\frac{0}{1}}{3}=\frac{0}{3}=0$$

$$\lim_{x\to0}\frac{\ln(\sin(2x))}{\ln(\sin(3x))}=\mathit{neexistuje}\text{, pouze pro } x\to0+=1$$

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$$\lim_{x\to0}x^3e^{-x}=0.1=0$$

$$\lim_{x\to0}x.\cot g(x)=\lim_{x\to0}\frac{x}{\frac{1}{\frac{\cos(x)}{\sin(x)}}}=\lim_{x\to0}\frac{x}{\frac{\sin(x)}{\cos(x)}}=\lim_{x\to0}\frac{1}{\frac{1}{\cos^2(x)}}=\cos^2(x)=1$$

$$\lim_{x\to\infty}x\left(\pi-2arctg(x)\right)=$$

$$\lim_{x\to1}(1-x)tg\left(\frac{\pi x}{2}\right)=\lim_{y\to0}y.\frac{\sin\left(\frac{\pi}{2}-\frac{\pi}{2}y\right)}{\cos\left(\frac{\pi}{2}-\frac{\pi}{2}y\right)}=\lim_{y\to0}y.\frac{\cos\left(\frac{\pi}{2}y\right)}{\sin\left(\frac{\pi}{2}y\right)}=\lim_{x\to1}\cos\left(\frac{\pi}{2}y\right).\frac{\frac{\pi}{2}y}{\sin\left(\frac{\pi}{2}y\right)}.\frac{2}{\pi}=1.1.\frac{2}{\pi}=\frac{2}{\pi}$$

$$\lim_{x\to0+}x^a\ln(x)=$$

$$\lim_{x\to-\infty}x^ne^{-x}=$$

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a) $\lim_{x\to0+}x^x=1$

b)

$$\lim_{x\to1}x^{\frac{1}{1-x}}=\lim_{x\to1}e^{\frac{1}{1-x}.\ln(x)}=e^{-1}=\frac{1}{e}$$

$$\lim_{x\to1}\frac{1}{1-x}.\ln(x)=\lim_{x\to1}\frac{\ln(x)}{1-x}=\frac{\frac{1}{x}}{-1}=-1$$

c)

$$\lim_{x\to\frac{\pi}{2}}\sin(x)^{\tan(x)}=\lim_{x\to\frac{\pi}{2}}e^{\tan(x).\ln(\sin(x))}=e^0=1$$

$$\lim_{x\to\frac{\pi}{2}}\tan(x).\ln(\sin(x))=\lim_{x\to\frac{\pi}{2}}\frac{\sin(x)}{\cos(x)}.\ln(x)=\lim_{x\to\frac{\pi}{2}}\frac{\cos(x).\ln(\sin(x))+\sin(x).\frac{\cos(x)}{\sin(x)}}{-\sin(x)}=$$

$$=\lim_{x\to\frac{\pi}{2}}-\frac{\cos(x)}{\sin(x)}.\ln(x)=-\frac{0}{1}.0=0$$

d)

$$\lim_{x\to\infty} x^x = \lim_{x\to\infty} e^{x.\ln(x)} = \lim_{x\to\infty} e^x = \infty$$
$$\lim_{x\to\infty} x.\ln(x) = \infty$$

e)

$$\lim_{x\to\infty}\left(\cos\frac{1}{x}\right)^{x^2} = \lim_{x\to\infty} e^{x^2.\ln\left(\cos\frac{1}{x}\right)} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$\lim_{x\to\infty} x^2.\ln\left(\cos\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(\cos\frac{1}{x}\right)}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{1.\sin\left(\frac{1}{x}\right)}{x^{2.}\cos\left(\frac{1}{x}\right)}}{-\frac{2}{x^3}} = \lim_{x\to\infty} -\frac{\sin\left(\frac{1}{x}\right).\frac{1}{x}}{\frac{\frac{1}{x}.\cos\left(\frac{1}{x}\right)}{\frac{2}{x}}} = \lim_{x\to\infty}\frac{-1}{x.\cos\left(\frac{1}{x}\right)}.\frac{x}{2} =$$
$$= \lim_{x\to\infty}\frac{-1}{2.\cos\left(\frac{1}{x}\right)} = \frac{-1}{2.1} = -\frac{1}{2}$$

f)

$$\lim_{x\to 0+}\left(\cot g(x)\right)^{\sin(x)} = \lim_{x\to 0+}\frac{\cos(x)^{\sin(x)}}{\sin(x)^{\sin(x)}} = \frac{1^0}{0^0} = \frac{1}{1} = 1$$

str. 85 př. 5
Určete rovnice tečný a normály grafu funkce f v bodě A.
a)
$y = \sin x, A = [0, y]$

$$t: y = f(a) + f'(a)(x-a)$$
$$y = \sin(0) + \cos(0)(x-0)$$
$$y = x$$

$$n: y = f(a) - \frac{1}{f'(a)}(x-a)$$
$$y = \sin(0) - \frac{1}{\cos(0)}(x-0)$$
$$y = -x$$

Rovnice tečny je $y = x$ a rovnice normály je $y = -x$.

b)
$y = \ln x, A = [1, y]$

$$y_t = \ln(1) + \frac{1}{1}(x-1)$$
$$y = x - 1$$

$$y_n = \ln(1) - \frac{1}{1}(x-1)$$
$$y_n = 1 - x$$

Rovnice tečny je $y = x - 1$ a rovnice normály je $y_n = 1 - x$.

c)

$y = 2x^2 - 1, A = \left[\frac{1}{2}, y\right]$

$y_t = 2.\frac{1}{4} + \frac{4}{2}\left(x - \frac{1}{2}\right)$

$y_n = 2.\frac{1}{4} - \frac{1}{\frac{4}{2}}\left(x - \frac{1}{2}\right)$

$y_t = 2x - \frac{3}{2}$

$y_n = -\frac{1}{x} - \frac{1}{4}$

Rovnice tečny je $y_t = 2x - \frac{3}{2}$ a rovnice normály je $y_n = -\frac{1}{x} - \frac{1}{4}$.

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Určete Taylorův polynom řádu n a funkce f v bodě a.

a)

$f(x) = \frac{1}{1-x}, a = 0, n = 3$

$f'(x) = -1(1-x)^{-2}$

$f''(x) = +(1-x)^{-3}$

$f'''(x) = 6(1-x)^{-4}$

$f(x) = (1-a)^{-1} + \frac{(1-a)^{-2}}{1!}(x-a)^1 + \frac{2(1-a)^{-3}}{2!}(x-a)^2 + \frac{6(1-x)^{-4}}{3!}(x-a)^3$

$f_{(0)} = 1 + x + x^2 + x^3$

b)

$f(x) = \arctan(x), a = 0, n = 3$

$f'(x) = \frac{1}{1+x^2} = \left(1+x^2\right)^{-1}$

$f''(x) = -2x\left(1+x^2\right)^{-2}$

$f'''(x) = -2x\left(1+x^2\right)^{-2} - 2x\left(-2\left(1+x^2\right)^{-3}\right)$

$f(x) = \arctan(x) + \frac{\left(1+a^2\right)^{-1}}{1!}(x-a) + \frac{-2a\left(1+a^2\right)^{-2}}{2!}(x-a)^2 + \frac{8a^2\left(1+a^2\right)^{-3}}{3!}(x-a)^3$

$f(x) = x - \frac{2}{6}x^3 = x - \frac{1}{3}x^3$

c)

$f(x) = \ln(1+x), a = 0, n = 4$

$f'(x) = (1+x)^{-1}$

$f''(x) = -(1+x)^{-2}$

$f'''(x) = 2(1+x)^{-3}$

$f^{4\prime}(x) = -6(1+x)^{-4}$

$$f(x) = \ln(1+x) + \frac{1}{1!(1+x)}(x-a) - \frac{1}{2!(1+x)^2}(x-a)^2 + \frac{2}{3!(1+x)^3}(x-a)^3 - \frac{6}{4!(1+x)^4}(x-a)^4$$

$$f(x) = x - \frac{1x^2}{2} + \frac{1}{3}x^3 - \frac{1}{4}x^4$$