str. 71, př. 1

$$\left(3x^2-5x+1\right)'=6x-5$$

$$\left(2\sqrt{x}-\frac{1}{x}\right)'=\left(2x^{\frac{1}{2}}-x^{-1}\right)'=\frac{1}{\sqrt{x}}+\frac{2}{x^2}=\frac{\sqrt{x}}{x}+\frac{2}{x^2}$$

$$\left(\frac{1}{\sqrt[5]{x^3}}+\frac{2}{x^3}\right)'=\left(x^{-\frac{3}{5}}+2x^{-3}\right)'=-\frac{3}{5}x^{-\frac{8}{5}}-\frac{6}{x^4}=-\frac{3}{\sqrt[5]{x^8}}-\frac{6}{x^4}$$

$$\left(\frac{x}{x^3+1}\right)'=\frac{\left(x^3+1\right)-\left(3x^2.x\right)}{\left(x^3+1\right)^2}=-\frac{2x^3+1}{\left(x^3+1\right)^2}$$

$$\left(\left(x^2+1\right)^4\right)'=2x.4(x^2+1)^3=8x\left(x^2+1\right)^3$$

$$\left(\sqrt{x^3+1}\right)'=3x\frac{1}{2}\left(x^3+1\right)^{\frac{-1}{2}}=\frac{3x}{2\sqrt{x^3+1}}$$

str. 71, př. 2

$$\left(\sqrt[4]{x}\cos(x)\right)'=\frac{1}{4}x^{-\frac{3}{4}}.\sqrt[4]{x}\left(-\sin(x)\right)=\frac{1}{4\sqrt[4]{x^3}}-\sqrt[4]{x}\sin\left(x\right)$$

$$\left(e^x\sin\left(x\right)\right)'=e^x\cos\left(x\right)+e^x\sin\left(x\right)=e^x.\left(\sin\left(x\right)+\cos\left(x\right)\right)$$

$$\left(x^2e^x\cos\left(x\right)\right)'=2x.e^x.\cos\left(x\right)+x^2.e^x.\cos\left(x\right)+x^2.e^x.\left(-\sin\left(x\right)\right)=$$
$$=e^x(2x.\cos\left(x\right)+x^2\cos\left(x\right)-x^2\sin\left(x\right)$$

$$\left(x.\sin\left(x\right).\arctan\left(x\right)\right)'=\sin\left(x\right).\arctan\left(x\right)+x.\cos\left(x\right).\arctan\left(x\right)+\frac{x.\sin\left(x\right)}{x^2+1}$$

$$\left(\frac{e^x}{\sin\left(x\right)}\right)'=\frac{e^x.\sin\left(x\right)-e^x\left(-\cos\left(x\right)\right)}{\sin^2\left(x\right)}=\frac{e^x.\sin\left(x\right)+\cos\left(x\right)}{\sin^2\left(x\right)}$$

$$\left(\cot g\left(x\right)\right)'=\left(\frac{\cos\left(x\right)}{\sin\left(x\right)}\right)'=\frac{-\sin\left(x\right).\sin\left(x\right)-\cos\left(x\right).\left(\cos\left(x\right)\right)}{\sin^2\left(x\right)}=\frac{-1}{\sin^2\left(x\right)}$$

str. 72, př. 3

$$\left(\sin\left(2x+5\right)\right)'=\left(2x+5\right)'.\left(\sin(2x+5)\right)'=2.\cos\left(2x+5\right)$$

$$\left(e^{-3x+1}\right)'=-3e^{-3x+1}$$

$$\left(10^x\right)'=10^x.\ln 10$$

$$\left(\ln^2 x\right)'=\left(\ln x\right)'.\left(\ln^2 x\right)'=\frac{1}{x}.2\ln\left(x\right)=\frac{2}{x}\ln\left(x\right)$$

$$\left(\ln\left(\tan x\right)\right)'=\left(\tan x\right)'.\left(\ln\left(\tan x\right)\right)'=\frac{1}{\cos^2\left(x\right)}.\frac{\cos\left(x\right)}{\sin\left(x\right)}=\frac{1}{\cos\left(x\right).\sin\left(x\right)}$$

$$\left(\arccos\left(\frac{2x-1}{\sqrt{3}}\right)\right)' = \left(\frac{2x-1}{\sqrt{3}}\right)'.\left(\arccos\left(\frac{2x-1}{\sqrt{3}}\right)\right)' = \frac{2}{3}\sqrt{3}.\left(\frac{-1}{\sqrt{1-\left(\frac{2x-1}{\sqrt{3}}\right)}}\right) =$$

$$= \frac{-2\sqrt{3}}{\sqrt{9}.\sqrt{1-\frac{4x^2-4x+1}{3}}} = \frac{-2\sqrt{3}}{\sqrt{9-3(4x^2-4x+1)}} = \frac{-2\sqrt{3}}{\sqrt{-12x^2+12x+6)}}$$

$$\left(\sqrt{\ln^2(x+1)}\right)' = (x+1)'.\left(\ln(x+1)\right)'.\left(\ln^2(x+1)\right)'.\left(\sqrt{\ln^2(x+1)}\right)' =$$

$$= 1.\frac{1}{x+1}.2\ln(x+1).\frac{1}{2\sqrt{\ln^2(x+1)}} = \frac{\ln(x+1)}{\sqrt{\ln^2(x+1)}.(x+1)}$$

$$\left(\ln\cosh(x)\right)' = \left(\cosh(x)\right)'.\left(\ln\cosh(x)\right)' = \sinh(x).\frac{1}{\cosh(x)} = \tanh(x)$$

$$\left(\ln\ln\sin(x)\right)'$$

Aby byla funkce definována musí být x>1 a D(x) pro sin(x) je $\langle -1;1\rangle$.

str. 72, př. 4

$$\left(x^x\right)' = \left(e^{x.\ln(x)}\right) = e^{x.\ln(x)}.\left(\ln(x)+x.\frac{1}{x}\right) = x^x.\left(\ln(x)+1\right)$$

$$\left(x^{\sin(x)}\right)' = \left(x^{\sin(x)}\right).\left(\frac{(x)'.\sin(x)}{x} + \left(\sin(x)\right)'.\ln(x)\right) = x^{\sin(x)}.\left(\frac{\sin(x)}{x} - \cos(x)\ln(x)\right)$$

$$\left(x^{x^2}\right)' = \left(e^{x^2.\ln(x)}\right)' = x^{x^2}.\left(2x\ln(x)+x^2\frac{1}{x}\right) = x^{x^2}x\left(2\ln(x)+1\right) = x^{x^2+1}\left(2\ln(x)+1\right)$$

$$\left(\left(\frac{x}{x+1}\right)^x\right)' = \left(\frac{x}{x+1}\right)^x.\left(\frac{\frac{1}{(x+1)^2}.x}{\frac{x}{x+1}}.1\ln\left(\frac{x}{x+1}\right)\right) = \left(\frac{x}{x+1}\right)^x.\left(\frac{1}{x+1}+\ln\left(\frac{x}{x+1}\right)\right)$$

$$\left(\left(x^2+1\right)^{\cos(\pi x)}\right)' = \left(x^2+1\right)^{\cos(\pi x)}.\left(\frac{2x.\cos(\pi x)}{x^2+1} + \left(-\sin(\pi x).\ln\left(x^2+1\right)\right)\right)$$

str. 72, př. 5

a)

$$y = x.e^{x^2}$$

$$y' = 1.e^{x^2} + x.\left(e^{x^2}.2x\right) = e^{x^2}.\left(1+2x^2\right)$$

$$y'' = \left(e^{x^2}.\left(1+2x^2\right)\right)' = e^{x^2}.2x.\left(1+2x^2\right) + e^{x^2}(4x) = e^{x^2}\left(2x+4x^3+4x\right) = 2e^{x^2}\left(2x^3+3x\right)$$

b)

$$y = \left(x^2+1\right) arctg\left(x\right)$$

$$y' = 2x.arctg\left(x\right) + \left(x^2+1\right).\frac{1}{1+x^2} = 2x.arctg\left(x\right)+1$$

$$y'' = \left(2x.arctg\left(x\right)+1\right)' = 2.arctg\left(x\right) + \frac{2x}{x^2+1}$$

$$u'.v.w + u.v'.w + u.v.w' =$$
$$= u'.v.1 + u.v'.1 + u.v.0 \Rightarrow$$
$$\Rightarrow u'.v + u'v$$

c)

$$\ln\left(x+\sqrt{x^2+1}\right)$$

str. 72, př. 6

a)

$$y = e^{ax}$$
$$y' = e^{ax}.a$$
$$y'' = e^{ax}.a^2$$
$$y''' = e^{ax}.a^3$$
$$y^{n'} = e^{ax}.a^n$$

b)

$$y = xe^x$$
$$y' = e^x + x.e^x = e^x\left(1+x\right)$$
$$y'' = e^x.1 + e^x\left(1+x\right) = e^x\left(1+1+x\right) = e^x\left(2+x\right)$$
$$y''' = e^x\left(2+x\right) + e^x.1 = e^x\left(2+x+1\right) = e^x\left(3+x\right)$$
$$y^{n'} = e^x\left(n+x\right)$$

c)

$$y = x.\ln\left(x\right)$$
$$y' = \ln\left(x\right) + x.\frac{1}{x} = \ln\left(x\right)+1$$
$$y'' = \frac{1}{x} = x^{-1}$$
$$y''' = -x^{-2}$$
$$y'''' = 2x^{-3}$$
$$y^{n'} = \left(-1\right)^n.\frac{(n-2)!}{x^{n-1}} = \left(-1\right)^n.\left(n-2\right)!.x^{1-n}$$