str. 22, př. 3(a-f)

$$\lim_{n\to\infty}\left(3n^3-2n^2+4n-1\right)=\lim_{x\to\infty}n^3\left(3-\frac{2}{n}+\frac{4}{n^2}-\frac{1}{n^3}\right)=\infty.(3-0+0+0)=+\infty$$

$$\lim_{n\to\infty}(n^4+n^3-5n^2)=\lim_{x\to\infty}n^4\left(1+\frac{1}{n}-\frac{5}{n^2}\right)=\infty(1+0-0)=+\infty$$

$$\lim_{n\to\infty}(-n^2+6n+2)=\lim_{x\to\infty}(-n^2)\left(1-\frac{6}{n}-\frac{2}{n^2}\right)=-\infty(1-0-0)=-\infty$$

$$\lim_{n\to\infty}(-4n^5+8n^4+n^3)=\lim_{x\to\infty}(-n^5)\left(4-\frac{8}{n}-\frac{1}{n^5}\right)=-\infty(4-0-0)=-\infty$$

$$\lim_{n\to\infty}(3^n-4.2^{n+3})=\lim_{x\to\infty}(3^n-4.2^n.8)=\lim_{x\to\infty}3^n\left(1-32.\left(\frac{2}{3}\right)^n\right)=+\infty(1-0)=\infty$$

$$\lim_{n\to\infty}(2.(-3)^{2n-1}-5.7^{n+2})=\textit{neexistuje}$$

$$\lim_{n\to\infty}(2.5^{n+1}+3.(-2)^{2n})=\lim_{x\to\infty}(10.5^n+3.4^n)=+\infty$$

$$\lim_{n\to\infty}(2.(-5)^{n-1}-5.2^{2n+2})=\textit{neexistuje}$$

str. 23, př. 4(a-f)

$$\lim_{n\to\infty}\frac{n^2-1}{n^3}=\lim_{x\to\infty}\frac{\frac{n}{n^3}-\frac{1}{n^3}}{\frac{n^3}{n^3}}=\lim_{x\to\infty}\frac{\frac{1}{n^2}-\frac{1}{n^3}}{1}=\frac{0-0}{1}=0$$

$$\lim_{n\to\infty}\frac{n+3}{n^2+2n-1}=\lim_{x\to\infty}\frac{\frac{n}{n^2}+\frac{3}{n^2}}{\frac{n^2}{n^2}+\frac{2n}{n^2}-\frac{1}{n^2}}=\lim_{x\to\infty}\frac{\frac{1}{n}+\frac{3}{n^2}}{1+\frac{2}{n}-\frac{1}{n^2}}=\frac{0+0}{1+0-0}=0$$

$$\lim_{n\to\infty}\frac{3n^3-2n^2+1}{n^3}=\lim_{x\to\infty}\frac{\frac{3n^3}{n^3}-\frac{2n^2}{n^3}+\frac{1}{n^3}}{\frac{n^3}{n^3}}=\lim_{x\to\infty}\frac{3-\frac{2}{n}+\frac{1}{n^3}}{1}=\frac{3-0+0}{1}=3$$

$$\lim_{n\to\infty}\frac{2n^2-3}{3n^2-n+4}=\lim_{x\to\infty}\frac{\frac{2n^2}{n^2}-\frac{3}{n^2}}{\frac{3n^2}{n^2}-\frac{n}{n^2}+\frac{4}{n^2}}=\lim_{x\to\infty}\frac{2-\frac{3}{n^2}}{3-\frac{1}{n}+\frac{4}{n^2}}=\frac{2-0}{3-0+0}=\frac{2}{3}$$

$$\lim_{n\to\infty}\frac{n^2+4n-1}{3n+1}=\lim_{x\to\infty}\frac{n(n+4)-1}{3n+1}=\lim_{x\to\infty}\frac{n+4-1}{3+\frac{1}{n}}=\frac{+\infty+4-1}{3+0}=+\infty$$

$$\lim_{n\to\infty}\frac{2-n^6}{1+2n^2}=\frac{n^6}{n^2}\left(\frac{\frac{2}{n^6}-1}{\frac{1}{n^2}+2}\right)=\lim_{x\to\infty}n^4.\left(\frac{0-1}{0+2}\right)=+\infty.\left(\frac{-1}{2}\right)=-\infty$$

str. 23, př. 5 (a-f)

$$\lim_{n\to\infty}\frac{2.3^n+(-1)^n}{4^n}=\lim_{x\to\infty}2.\left(\frac{3}{4}\right)^n+\lim_{x\to\infty}\left(\frac{-1}{4}\right)^n=2.0+0=0$$

$$\lim_{n\to\infty}\frac{3.2^n+(-3)^n}{3^n-2^{2n}}=\lim_{x\to\infty}\frac{(-3)^n.\left(1+3.\left(\frac{2}{-3}\right)^n\right)}{4^n\left(\left(\frac{3}{4}\right)^n-1\right)}=\lim_{x\to\infty}\left(\frac{-3}{4}\right)^n.\frac{\left(1+3.\left(\frac{2}{-3}\right)^n\right)}{\left(\left(\frac{3}{4}\right)^n-1\right)}=0.\left(\frac{1+0}{0-1}\right)=0$$

$$\lim_{n\to\infty}\frac{5.3^n-3.2^{n+2}}{2^n-2.3^n}\lim_{x\to\infty}\frac{5.3^n-3.2^n.4}{2^n-2.3^n}\lim_{x\to\infty}\frac{3^n}{3^n}.\left(\frac{5-12.\left(\frac{2}{3}\right)^n}{\left(\frac{2}{3}\right)^n-2}\right)=1.\left(\frac{5-12.0}{0-2}\right)=-\frac{5}{2}$$

$$\lim_{n\to\infty}\frac{2.(-5)^n+3.4^n}{(-5)^n+2^{n+3}}=\lim_{x\to\infty}\frac{(-5)^n}{(-5)^n}\left(\frac{2+3\left(\frac{4}{-5}\right)^n}{1+8.\left(\frac{2}{-5}\right)^n}\right)=1.\left(\frac{2+0}{1+0}\right)=2$$

$$\lim_{n\to\infty}\frac{3^{n+1}-(-4)^n}{2.3^n+2^{2n-1}}=\lim_{x\to\infty}\left(\frac{-4}{4}\right)^n.\left(\frac{\left(\frac{3}{-4}\right)^n.3-1}{2.\left(\frac{3}{4}\right)^n+\frac{1}{2}}\right)=\textit{neexistuje}$$

$$\lim_{x\to\infty}\frac{2.5^n+(-4)^n}{2^n+4^n}=\lim_{x\to\infty}\left(\frac{5}{4}\right)^n.\left(\frac{2+\left(\frac{-4}{5}\right)^n}{\left(\frac{2}{4}\right)^n+1}\right)=+\infty.\left(\frac{2.0}{0+1}\right)=+\infty$$

str. 23, př. 6 (a-e)

$$\lim_{x\to\infty}\left(\frac{n^5+1}{2n^5+1}\right)^4=\lim_{x\to\infty}\left(\frac{n^5}{n^5}\right)^4.\left(\frac{1+\frac{1}{n^5}}{2+\frac{1}{n^5}}\right)^4=1.\left(\frac{1+0}{2}\right)^4=\frac{1}{16}$$

$$\lim_{x\to\infty}\left(\frac{3n^2-n+5}{2n^3+5n-3}\right)^{-3}=\lim_{x\to\infty}\left(\frac{n^3}{n^2}\right)^3.\left(\frac{2+\frac{5}{n^2}-\frac{3}{n^3}}{3-\frac{1}{n}-\frac{3}{n^2}}\right)^3=\lim_{x\to\infty}n^3.\left(\frac{2+0-0}{3-0-0}\right)^3=+\infty$$

$$\lim_{x\to\infty}\frac{(n+2)^2+(n-1)^3}{(2n+1)^3-(n+1)^2}=\lim_{x\to\infty}\frac{\left(n^2+4n+4\right)+\left(n^3-3n^2+3n-1\right)}{\left(8n^3+12n^2+6n+1\right)-\left(n^2+2n+1\right)}=\lim_{x\to\infty}\frac{n^3-2n^2+7n+3}{8n^3+12n^2+4n}=$$

$$=\frac{n^3}{n^3}.\left(\frac{1-\frac{2}{n}+\frac{7}{n^2}+\frac{3}{n^3}}{8+\frac{12}{n}+\frac{4}{n^2}}\right)=1.\left(\frac{1-0+0}{8+0+0}\right)=\frac{1}{8}$$

$$\lim_{x\to\infty}\frac{\sqrt{n}+\sqrt[3]{n}}{n+\sqrt{n}}=\lim_{x\to\infty}\left(\frac{\frac{\sqrt{n}}{\sqrt{n}}+\frac{\sqrt[3]{n}}{\sqrt{n}}}{\frac{n}{\sqrt{n}}+\frac{\sqrt{n}}{\sqrt{n}}}\right)=\lim_{x\to\infty}\frac{1+\frac{1}{\sqrt[6]{n}}}{\sqrt{n}+1}=\frac{1+0}{\infty+1}=0$$

$$\lim_{x\to\infty}\frac{\sqrt{4n^2+n}-\sqrt{n^2-n}}{n}=\lim_{x\to\infty}\frac{\sqrt{4n^2+n}-\sqrt{n^2-n}}{n}.\frac{\sqrt{4n^2+n}+\sqrt{n^2-n}}{\sqrt{4n^2+n}+\sqrt{n^2-n}}=$$

$$=\lim_{x\to\infty}\frac{\left(4n^2+n\right)-\left(n^2-n\right)}{n.\left(\sqrt{4n^2+n}+\sqrt{n^2-n}\right)}=\lim_{x\to\infty}\frac{n(3n+2)}{n.\left(\sqrt{4n^2+n}+\sqrt{n^2-n}\right)}=\lim_{x\to\infty}\frac{n}{n}.\frac{\left(3+\frac{2}{n}\right)}{\sqrt{4+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}}=1.\frac{3+0}{\sqrt{4+0}+\sqrt{1-0}}=\frac{3}{2+1}=\frac{3}{3}=1$$

str. 23, př. 7 (a-d)

$$\lim_{x\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right)=\lim_{x\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right).\frac{\left(\sqrt{n+1}+\sqrt{n}\right)}{\left(\sqrt{n+1}+\sqrt{n}\right)}=\lim_{x\to\infty}\frac{\sqrt{n}\left(n+1-n\right)}{\sqrt{n+1}+\sqrt{n}}=\lim_{x\to\infty}\frac{\frac{\sqrt{n}}{\sqrt{n}}+\frac{1}{\sqrt{n}}}{\sqrt{\frac{n}{n}+\frac{1}{n}}+\frac{\sqrt{n}}{\sqrt{n}}}=\frac{1+0}{\sqrt{1+0}+\sqrt{1}}=\frac{1}{2}$$

$$\lim_{x\to\infty}\sqrt{n^2+4n}-n=\lim_{x\to\infty}\left(\sqrt{n^2+4n}-n\right).\left(\frac{\sqrt{n^2+4n}+n}{\sqrt{n^2+4n}+n}\right)=\lim_{x\to\infty}\frac{n^2+4n-n^2}{\sqrt{n^2+4n}+n}=\lim_{x\to\infty}\frac{4}{\sqrt{\frac{n^2}{n^2}+\frac{4}{n}}+1}=\frac{4}{\sqrt{1+0}+1}=\frac{4}{2}=2$$

$$\lim_{x\to\infty}(n-2)\left(\sqrt{4n^2+3}-2n\right)=\lim_{x\to\infty}(n-2)\left(\sqrt{4n^2+3}-2n\right).\left(\frac{\sqrt{4n^2+3}+2n}{\left(\sqrt{4n^2+3}+2n\right)}\right)=\lim_{x\to\infty}\frac{(n-2)\left(4n^2+3-4n^2\right)}{\sqrt{4n^2+3}+2n}=\lim_{x\to\infty}\frac{3n-6}{\sqrt{4n^2+3}+2n}=$$

$$=\lim_{x\to\infty}\frac{3-\frac{6}{n}}{\sqrt{4+\frac{3}{n^2}}+2}=\frac{3-0}{\sqrt{4+0}+2}=\frac{3}{4}$$

$$\lim_{x\to\infty}\sqrt{n}\left(\sqrt{n^3+2n}-\sqrt{n^3-1}\right)=\lim_{x\to\infty}\sqrt{n}\left(\sqrt{n^3+2n}-\sqrt{n^3-1}\right).\frac{\left(\sqrt{n^3+2n}+\sqrt{n^3-1}\right)}{\left(\sqrt{n^3+2n}+\sqrt{n^3-1}\right)}=\lim_{x\to\infty}\frac{\sqrt{n}\left(\left(n^3+2n\right)-\left(n^3-1\right)\right)}{\sqrt{n^3+2n}+\sqrt{n^3-1}}=$$

$$=\lim_{x\to\infty}\frac{\sqrt{n}.\left(2n-1\right)}{\sqrt{n^3+2n}+\sqrt{n^3-1}}=\lim_{x\to\infty}\frac{2n\sqrt{n}+\sqrt{n}}{\sqrt{n^3+2n}+\sqrt{n^3-1}}=\lim_{x\to\infty}\frac{2+\frac{1}{\sqrt[5]{n}}}{\sqrt{\frac{n^3}{n^3}+\frac{2}{n^2}}+\sqrt{1-\frac{1}{n^3}}}=\frac{2+0}{\sqrt{1+0}+\sqrt{1-0}}=\frac{2}{2}=1$$