46/1 $x' = \frac{e^{-x}}{t}$ $x(1)=0$ $t \in (-\infty; 0) \cup (0; +\infty)$

$\frac{dx}{dt} = \frac{e^{-x}}{t}$

$\int e^{x} dx = \int \frac{1}{t} dt$

$e^{x} = \ln|t| + \ln C$

$x(t) = \ln|\ln(ct)|$ $t \in (0; +\infty)$

$e^{0} = \ln|1| + \ln C$

$1 = 0 + \ln C$

$\ln C = 1$

$c = e$

$e^{x} = \ln|t| + \ln e$

$e^{x} = \ln|t| + 1$

$\boxed{x = \ln(\ln|t| + 1); \quad t \in (\frac{1}{e}; +\infty)}$

46/2 $x' = x^{-2}$ $t \in \mathbb{R}$ $x \in (-\infty; 0) \cup (0; +\infty)$

$\frac{dx}{dt} = \frac{1}{x^2}$

$\int x^2 dx = \int dt$

$\frac{1}{3} x^3 = t + c$

$x^3 = 3(t + c)$

$x(t) = \sqrt[3]{3(t+c)}$

a) $x(1) = 1$

$1 = 3.(1 + c)$

$1 = 3 + 3c$

$c = -\frac{2}{3}$

$\boxed{x(t) = \sqrt[3]{3(t - \frac{2}{3})} = \sqrt[3]{3t - 2} \quad ; t \in (\frac{2}{3}; +\infty)}$

b) $x(-2) = 1$

$(1)^3 = 3(-2 + c)$

$1 = -6 + 3c$

$7 = 3c$

$c = \frac{7}{3}$

$\boxed{x(t) = \sqrt[3]{3(t + \frac{7}{3})} = \sqrt[3]{3t + 7} \quad t \in (-\frac{7}{3}; +\infty)}$

c) $x(-2) = -2$

$(-2)^3 = 3(-2 + c)$

$-8 = -6 + 3c$

$-2 = 3c$

$c = \frac{-2}{3}$

$\boxed{x(t) = \sqrt[3]{3(t - \frac{2}{3})} = \sqrt[3]{3t - 2} \quad t \in (-\infty; \frac{2}{3})}$

46/3 $\quad x' = \frac{-t}{x} \qquad x \neq 0 \; ; t \in \mathbb{R}$

$$\frac{dx}{dt} = \frac{-t}{x}$$

$$\int x\,dx = -\int t\,dt$$

$$\frac{1}{2}x^2 = -\frac{1}{2}t^2 + c$$

$$x^2 = -t^2 + c$$

$$x(t) = \pm\sqrt{c - t^2}$$

a) $x(1) = 1$

$$1 = -1 + c$$

$$c = 2$$

$$\boxed{x(t) = \sqrt{2 - t^2} \quad ; t \in (-\sqrt{2}; \sqrt{2})}$$

b) $x(4) = -3$

$$(-3)^2 = -(4)^2 + c$$

$$9 = -16 + c$$

$$c = 25$$

$$\boxed{x(t) = -\sqrt{25 - t^2} \; ; \; t \in (-5; 5)}$$

46/4 $\quad x' = -x^2 \qquad x \neq 0$

$$\frac{dx}{dt} = -1 \cdot x^2$$

$$\int \frac{1}{x^2}\,dx = -\int dt$$

$$-\frac{1}{x} = -t + c$$

$$x(t) = \frac{1}{t - c}$$

a) $x(-1) = 0$

$$\boxed{x(t) = 0}$$

b) $x(1) = 3$

$$3 = \frac{1}{1 - c}$$

$$3 - 3c = 1$$

$$c = \frac{2}{3}$$

$$\boxed{x(t) = \frac{3}{3t - 2} \quad ; \; t \in \left(\frac{2}{3}; +\infty\right)}$$

c) $x(-2) = -1$

$$-1 = \frac{1}{-2 - c}$$

$$-2 - c = -1$$

$$c = -1$$

$$\boxed{x(t) = \frac{1}{t + 1} \quad ; \; t \in (-\infty; -1)}$$

46/5 $x' = \frac{(x^2 - x)}{t}$ $t \in (-\infty; 0) \cup (0; \infty)$

$\frac{dx}{dt} = \frac{(x^2 - x)}{t}$

$\int \frac{1}{x.(x-1)} = \int \frac{1}{t} dt$

$\int \left(\frac{1}{x} - \frac{1}{x-1} \right) dx = \int \frac{1}{t} dt$

$\ln \left| \frac{x}{x-1} \right| = \ln t + \ln C$

$\frac{x}{x-1} = ct$

$1 - \frac{1}{x-1} = ct$

$1 - ct = \frac{1}{x-1}$

$\frac{1}{1-ct} = x - 1$

$x(t) = \frac{1}{1-ct} + 1$

$\frac{1}{x.(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$A = \frac{1}{x-1} \Big|_{x=0} = -1$

$B = \frac{1}{x} \Big|_{x=1} = 1$

$x : (x-1) = 1 - \frac{1}{x-1}$

$-x+1$

?

a) $x(t) = \frac{2}{2-t}$ $t \in (0; 2)$

b) $x(t) = 1$; $t \in (-\infty; 0)$

c) $x(t) = \frac{3}{3+t}$; $t \in (0; +\infty)$

d) $x(t) = 0$; $t \in (0; \infty)$

e) $x(t) = \frac{1}{1-2t}$ $t \in \left(\frac{1}{2}; +\infty \right)$

46/6 $\qquad x' = \dfrac{1-x^2}{2+t}$

$$\frac{dx}{dt} = \frac{1-x^2}{2+x}$$

$$+\int \frac{-2x}{1-x^2}\,dx = -\int \frac{1}{t}\,dt$$

$$\ln|1-x^2| = -\ln|t| + \ln c$$

$$1-x^2 = \frac{c}{t}$$

$$x(t) = \pm\sqrt{1-\frac{c}{t}}$$

a) $x(1) = \frac{1}{2}$; $\qquad x(t) = \sqrt{1-\frac{3}{4t}}$ $\quad ; t \in \left(\frac{3}{4}; +\infty\right)$

b) $x(-2) = 1$ $\quad ; x(t) = 1$ $\qquad t \in (-\infty; 0)$

c) $x(2) = 2$ $\quad ; x(t) = \sqrt{1+\frac{6}{t}}$ $\qquad t \in (0; +\infty)$

d) $x(3) = -2$ $\qquad x(t) = -\sqrt{1+\frac{9}{t}}$ $\qquad t \in (0; +\infty)$

e) $x(-3) = -\frac{2}{3}$ $\qquad x(t) = -\sqrt{1+\frac{5}{3}t}$ $\qquad t \in \left(-\infty; -\frac{5}{3}\right)$

46/7 $\qquad x' = 2\sqrt{x}$

$$\frac{dx}{dt} = 2\sqrt{x}$$

$$\int x^{-\frac{1}{2}}\,dx = 2\int t\,dt$$

$$\frac{1}{2}\sqrt{x} = \frac{2}{2}t^2 + c$$

$$\sqrt{x} = 2t^2 + c$$

$$x(t) = (t+c)^2$$

a) $x(0) = 1$

$x_t = 0 \qquad t \in (-\infty; -1\rangle$

$x_t = (t+1)^2 \quad t \in (-1; +\infty)$

b) $x(t) = 0 \qquad t \in \mathbb{R}$

$x(t) = 0 \qquad t \in (-\infty; c\rangle$

$x(t) = (t+c)^2 \quad t \in \langle -c; +\infty)$

46/8a) $x' = \frac{2tx}{t^2-1}$ $x(3) = 4$ $x \in (-\infty;-1) \cup (-1;1) \cup (1;\infty)$

$$\frac{dx}{dt} = \frac{2tx}{t^2-1}$$

$$\int \frac{1}{x}\,dx = \int \frac{2t}{t^2-1}\,dt$$

$$\ln|x| = \ln|t^2-1| + \ln c$$

$$x(t) = c\cdot(t^2-1) \quad - \text{OŘ}$$

$$\boxed{x(t) = \frac{1}{2}\cdot(t^2-1)} \quad t \in (1;\infty)$$

$x(t) = c\cdot(t^2-1)$

$4 = c\cdot(3^2-1)$

$4 = c\cdot 8$

$c = \frac{1}{2}$

46/8b) $x' = \frac{x+2}{t}$; $x(2) = 4$ $t \in (-\infty;0) \cup (0;+\infty)$

$$\frac{dx}{dt} = \frac{x+2}{t}$$

$$\int \frac{1}{x+2}\,dx = \int \frac{1}{t}\,dt$$

$$\ln|x+2| = \ln|t| + \ln c$$

$$x+2 = ct$$

$$x(t) = ct - 2 \quad - \text{OŘ}$$

$$\boxed{x(t) = 3t - 2 \quad ; t \in (0;+\infty)}$$

$x(t) = ct - 2$

$4 = 2c - 2$

$c = 3$

46/8c) $x' = \frac{2x+4}{t}$; $x(1) = 3$ $t \in (-\infty;0) \cup (0;+\infty)$

$$\frac{dx}{dt} = \frac{2(x+2)}{t}$$

$$\int \frac{1}{x+2}\,dx = 2\int \frac{1}{t}\,dt$$

$$\ln|x+2| = 2\ln|t| + \ln c$$

$$x+2 = ct^2$$

$$x(t) = ct^2 - 2 \quad - \text{OŘ}$$

$x(t) = ct^2 - 2$

$3 = c - 2$

$c = 5$

$$\boxed{x(t) = 5t^2 - 2 \quad ; t \in (0;+\infty)}$$

46/8d) $x' = \frac{-x}{t+1}$ $x(1) = 2$ $t \in (-\infty;-1) \cup (-1;+\infty)$

$\frac{dx}{dt} = \frac{-x}{t+1}$

$\int \frac{1}{x} dx = -\int \frac{1}{t+1} dt$

$\ln|x| = -\ln|t+1| + \ln c$

$x(t) = \frac{c}{t+1}$ - $O\overline{R}$

$x(t) = \frac{c}{t+1}$

$2 = \frac{c}{1+1}$

$c = 4$

$\boxed{x(t) = \frac{4}{t+1} \; ; \; t \in (-1; \infty)}$

46/8e) $x' = \frac{-3 \cdot (x+1)}{t}$; $x(1) = 0$ $t \in (-\infty;0) \cup (0;+\infty)$

$\frac{dx}{dt} = \frac{-3(x+1)}{t}$

$\int \frac{1}{x+1} dx = -3 \int \frac{1}{t} dt$

$\ln|x+1| = -3 \ln|t| + \ln c$

$x+1 = c t^{-3}$

$X(t) = c t^{-3} - 1$ $-O\overline{R}$

$x(t) = c t^{-3} - 1$

$0 = c \cdot 1 - 1$

$c = +1$

$\boxed{x(t) = t^{-3} + 1 \; ; \; t \in (0; \infty)}$

46/8f) $x' = (x-1) \cos t$ $x(\pi) = 0$ $t \in R$

$\frac{dx}{dt} = (x-1) \cdot \cos t$

$\int \frac{1}{x-1} dx = \int \cos t \, dt$

$\ln|x-1| = \sin t + c$

$x - 1 = e^{\sin t + c}$

$X(t) = c \cdot e^{\sin t} + 1$ $-O\overline{R}$

$x(t) = c \cdot e^{\sin t} + 1$

$0 = c \cdot e^{0} + 1$

$c = -1$

$\boxed{x(t) = 1 - e^{\sin t} \; ; \; t \in R}$

4P/68 g) $x' = -2t(x+1)$ $\quad x(0)=2$ $\quad t \in R$

$$\frac{dx}{dt} = -2t(x+1)$$
$$\int \frac{1}{x+1}\,dx = -2\int t\,dt$$
$$\ln|x+1| = -t^2 + c$$
$$x+1 = e^{c-t^2}$$
$$x(t) = c\cdot e^{-t^2} - 1 \quad \text{- OR}$$
$$\boxed{x(t) = 3e^{-t^2} - 1 \ ; \ t \in R}$$

$$x(t) = c\cdot e^{-t^2} - 1$$
$$2 = c\cdot 1 - 1$$
$$c = 3$$

4P/8 h) $x' = (x-1)\cdot \operatorname{cotg} t$ $\quad x(\frac{\pi}{2}) = 3$ $\quad t \in (0;\pi)$

$$\frac{dx}{dt} = (x-1)\cdot \operatorname{cotg} t$$
$$\int \frac{1}{(x-1)}\,dx = \int \frac{\cos t}{\sin t}\,dt$$
$$\ln|x-1| = \ln|\sin t| + \ln c$$
$$x-1 = c\cdot \sin t$$
$$x(t) = c\cdot \sin t + 1 \quad \text{- OR}$$
$$\boxed{x(t) = 2\sin t + 1 \ ; \ t \in (0;\pi)}$$

$$x(t) = c\cdot \sin t + 1$$
$$3 = c\cdot \sin\frac{\pi}{2} + 1$$
$$3 = c\cdot 1 + 1$$
$$c = 2$$

4P/8 i) $x' = -\frac{tx}{t+1}$ $\quad x(0)=2$ $\quad t \in (-\infty;-1)\cup(-1;\infty)$

$$\frac{dx}{dt} = \frac{-tx}{t+1}$$
$$\int \frac{1}{x}\,dx = -\int \frac{t}{t+1}\,dt$$
$$\ln|x| = -\left[t - \ln|t+1|\right] + \ln c$$
$$\ln|x| = \ln|t+1| - t + \ln c$$
$$x(t) = c\,(t+1)\cdot e^{-t}$$
$$\boxed{x(t) = 2(t+1)\cdot e^{-t} \ ; \ t \in (-1;\infty)}$$

$$x(t) = c\cdot(t+1)\cdot e^{-t}$$
$$2 = c\cdot(0+1)\cdot e^{0}$$
$$2 = c$$

4P/8 j) $x' = \frac{tx}{t+1}$ $\quad x(0)=1$ $\quad t \in (-\infty;-1)\cup(-1;\infty)$

$$\frac{dx}{dt} = \frac{tx}{t+1}$$
$$\int \frac{1}{x}\,dx = \int \frac{t}{t+1}\,dt$$
$$\ln|x| = t - \ln|t+1| + \ln c$$
$$x(t) = \frac{c}{t+1}\cdot e^{t}$$
$$\boxed{x(t) = \frac{e^{t}}{t+1} \quad t \in (-1;\infty)}$$

$$x(t) = \frac{c}{t+1}\cdot e^{t}$$
$$1 = \frac{c}{0+1}\cdot e^{0}$$
$$c = 1$$

47/9a)

$x' = \frac{1}{t}x + \frac{1}{t^3}$ $\quad x(1) = 1$ $\qquad t \in (-\infty; 0) \cup (0; +\infty)$

$\frac{dx}{dt} = \frac{x}{t}$

$\int \frac{1}{x} dx = \int \frac{1}{t} dt$

$\ln|x| = \ln|t| + \ln C$

$\tilde{x} = ct$

$\tilde{x}' = c't + c(t)' = c't + c$ $\quad (t)' = 1$

$c't + \cancel{c} = \frac{1}{\cancel{t}} \cancel{c t} + \frac{1}{t^3}$

$c't = t^{-3}$

$c' = t^{-4}$

$c = -\frac{1}{3}t^{-3} + k$

$\hat{x} = c \cdot t = \left(-\frac{1}{3}t^{-3}\right) \cdot t = -\frac{1}{3}t^{-2}$

$x(t) = \tilde{x} + \hat{x} = ct - \frac{1}{3}t^{-2}$ - obecné řešení

$x(1) = 1$

$1 = c \cdot 1 - \frac{1}{3} \cdot 1^{-2}$

$1 = c - \frac{1}{3}$

$3 = 3c - 1$

$c = \frac{4}{3}$

$x(t) = ct - \frac{1}{3}t^{-2}$

$\boxed{x(t) = \frac{4}{3}t - \frac{1}{3}t^{-2} = \frac{1}{3}\left(4t - t^{-2}\right); \ t \in (0; +\infty)}$

47/9b $\quad x' = -\frac{1}{t}x + \frac{1}{t^2}$; $x(1) = 2$

$\frac{dx}{dt} = -\frac{1}{t}x$

$\int \frac{1}{x} dx = -\int \frac{1}{t} dt$

$\ln|x| = -\ln|t| + \ln C$

$\tilde{x} = \frac{c}{t}$ $\qquad c \neq 0$ - (nemělo by smysl)

$\hat{x}' = \frac{c't - c(t)'}{t^2} = c't^{-1} - ct^{-2}$ $\quad (t)' = 1$

$c't^{-1} \cancel{- ct^{-2}} = \cancel{-\frac{1}{t}\frac{c}{t}} + \frac{1}{t^2}$

$c't^{-1} = t^{-2}$

$c' = t^{-1}$

$c = \ln|t| + k$

$\hat{x} = \frac{\ln|t|}{t}$

$x(t) = \tilde{x} + \hat{x} = \frac{c}{t} + \frac{\ln|t|}{t} = \frac{c + \ln|t|}{t}$ - obecné řešení

$t(1) = 2$

$2 = \frac{c + 0}{1}$

$c = 2$

$\boxed{x(t) = \frac{2 + \ln|t|}{t} = (\ln|t| + 2) \cdot t^{-1} \qquad t \in (0; +\infty)}$

47/9c $\qquad t \in (-\infty;0) \cup (0;+\infty)$

$x' = \frac{2}{t}x + t^2 \sin t \; ; \quad x(\pi) = 0$

$\tilde{x}' = c't^2 + 2ct$

$c't^2 + \cancel{2ct} = \cancel{\frac{2}{t} \cdot ct^2} + t^2 \sin t$

$c' \cancel{t^2} = \cancel{t^2} \sin t$

$c' = \sin t$

$c = -\cos t + k$

$\frac{dx}{dt} = \frac{2x}{t}$

$\int \frac{1}{x} dx = 2 \int \frac{1}{t} dt$

$\ln|x| = 2 \ln|t| + \ln c$

$\tilde{x} = ct^2$

$\hat{x} = c \cdot t^2 = -\cos t \cdot t^2$

$x(t) = ct^2 - t^2 \cos t = t^2 \cdot (c - \cos t)$ – obecné řešení

$x(\pi) = 0$

$0 = c \cdot \pi^2 - \pi^2 \cdot (-1)$

$0 = c \cdot \pi^2 + \pi^2$

$0 = (c+1) \cdot \pi^2 \quad / : \pi^2$

$c = -1$

$\boxed{x(t) = -t^2 (1 + \cos t) \; ; \; t \in (0;+\infty)}$

47/9d $\qquad x' = -\frac{2}{t}x + 4t \qquad x(2) = 5 \; ; \quad t \in (-\infty;0) \cup (0;+\infty)$

$\frac{dx}{dt} = -\frac{2}{t}x$

$\int \frac{1}{x} dx = -2 \int \frac{1}{t} dt$

$\ln|x| = -2 \ln|t| + \ln c$

$\tilde{x} = ct^{-2}$

$\hat{x} = t^2$

$\tilde{x}' = c't^{-2} - 2ct^{-3}$

$c't^{-2} \cancel{- 2ct^{-3}} = \cancel{-\frac{2}{t} ct^{-2}} + 4t$

$c't^{-2} = 4t$

$c' = 4t^3$

$c = t^4 + k$

$x(t) = \tilde{x} + \hat{x} = ct^{-2} + t^2 \quad ; \; x(2) = 5$ – oř

$5 = c \cdot (2)^{-2} + 4$

$5 = \frac{c}{4} + 4$

$20 = c + 16$

$4 = c$

$\boxed{x(t) = 4t^{-2} + t^2 \quad ; \; t \in (0;+\infty)}$

47) 9e $\quad x' = \frac{3}{t}x - t^3 e^t \quad x(1)=0 \quad t \in (-\infty;0) \cup (0;+\infty)$

$$\frac{dx}{dt} = \frac{3}{t}x$$

$$\int \frac{1}{x}dx = 3\int \frac{1}{t}dt$$

$$\ln|x| = 3\ln|t| + c$$

$$\tilde{x} = ct^3$$

$$\hat{x} = e^t t^3$$

$$\tilde{x}' = c't^3 + 3ct^2$$

$$c't^3 + \cancel{3ct^2} = \cancel{\frac{3}{t}\cdot ct^3} - t^3 e^t$$

$$c't^3 = -t^3 e^t$$

$$c' = -e^t$$

$$c = e^t + k$$

$x(t) = \tilde{x} + \hat{x} = ct^3 + e^t t^3 = t^3\cdot(c+e^t)$; ~~obecné~~

$x(1) = 0$

$$0 = t^3\cdot(c+e^t)$$

$$0 = 1\cdot(c+e^1)$$

$$-e = c$$

$$\boxed{x(t) = t^3\cdot(e^t - e) \ ; \ t \in (0;+\infty)}$$

47/9 f $\quad x' = -\frac{3}{t}x + \frac{2}{t^2} \quad x(1) = 3 \quad t \in (-\infty;0)\cup(0;+\infty)$

$$\frac{dx}{dt} = -\frac{3}{t}x$$

$$\int \frac{1}{x}dx = -3\int \frac{1}{t}dt$$

$$\ln|x| = -3\ln|t| + \ln c$$

$$\tilde{x} = ct^{-3}$$

$$\hat{x} = t^{-1}$$

$$\tilde{x}' = c't^{-3} - 3ct^{-4}$$

$$c't^{-3} - \cancel{3ct^{-4}} = \cancel{-\frac{3}{t}ct^{-3}} + 2t^{-2}$$

$$c't^{-3} = 2t^{-2}$$

$$c' = 2t$$

$$c = t^2 + k$$

$x(t) = \tilde{x} + \hat{x} = t^{-1}\cdot(ct^{-2}+1)$ – OŘ) $\quad x(1) = 3$

$$3 = 1^{-1}\cdot(c\cdot 1^{-2}+1)$$

$$3 = c+1$$

$$c = 2$$

$$\boxed{x(t) = t^{-1}\cdot(2t^{-2}-1) \ ; \ t \in (0;+\infty)}$$

47/9 g) $x' = 2tx - e^{t^2}$ $x(0) = 2$ $t \in \mathbb{R}$

$$\frac{dx}{dt} = 2tx$$

$$\int \frac{1}{x}dx = 2\int t\,dt$$

$$\ln|x| = t^2 + c$$

$$\tilde{x} = e^{t^2+c}$$

$$\tilde{x} = c \cdot e^{t^2}$$

$$\tilde{x}' = c' e^{t^2} + c \cdot 2t \cdot e^{t^2}$$

$$c' e^{t^2} + \cancel{2cte^{t^2}} = \cancel{2t \cdot c \cdot e^{t^2}} - e^{t^2}$$

$$c' e^{t^2} = -e^{t^2}$$

$$c' = -1$$

$$c = -t + k$$

$$\hat{x} = -te^{t^2}$$

$$x(t) = c \cdot e^{t^2} - e^{t^2} = e^{t^2} \cdot (c - t) \quad - \overline{\mathrm{OR}} \quad ; x(0) = 2$$

$$2 = e^0 \cdot (c - 0)$$

$$2 = c$$

$$\boxed{x(t) = e^{t^2}(2 - t) \; ; t \in \mathbb{R}}$$

47/9 h) $x' = -x \cdot tg\,t + \cos t$ $x(0) = 1$ $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\frac{dx}{dt} = -x \cdot tg\,t$$

$$\int \frac{1}{x}dx = +\int \frac{-\sin t}{\cos t}dt$$

$$\ln|x| = + \ln|\cos t| + \ln C$$

$$\tilde{x} = c \cdot \cos t$$

$$\tilde{x}' = c' \cos t - c \cdot \sin t$$

$$c' \cdot \cos t \cancel{- c \cdot \sin t} = \cancel{-c \cos t \cdot \frac{\sin t}{\cos t}} + \cos t$$

$$c' \cdot \cos t = \cos t$$

$$c' = 1$$

$$c = t + k$$

$$\hat{x} = t \cdot \cos t$$

$$x(t) = \tilde{x} + \hat{x} = c \cdot \cos t + t \cdot \cos t \quad - \overline{\mathrm{OR}} \quad x(0) = 1$$

$$1 = c \cdot 1 + 0 \cdot 1$$

$$c = 1$$

$$\boxed{x(t) = \cos t + t \cdot \cos t \; ; t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)}$$

47/9 i) $x' = \frac{1}{t+1} \cdot x + 1$ $\quad x(0) = -1 \quad t \in (-\infty; -1) \cup (-1; +\infty)$

$$\frac{dx}{dt} = \frac{x}{t+1}$$

$$\int \frac{1}{x}\,dx = \int \frac{1}{t+1}\,dt$$

$$\ln|x| = \ln|t+1| + \ln c$$

$$\tilde{x} = c(t+1)$$

$$\tilde{x}' = c'\cdot(t+1) + c\,(t+1)' \rightarrow 1$$

$$\tilde{x}' = c'\cdot(t+1) + c$$

$$c'\cdot(t+1) \cancel{+c} = \frac{1}{\cancel{t+1}}\cdot \cancel{c(t+1)} + 1$$

$$c' = \frac{1}{t+1}$$

$$c = \ln|t+1| + k$$

$$\hat{x} = (\ln|t+1|)\cdot(t+1)$$

$$x(t) = c\cdot(t+1) + (t+1)\cdot \ln|t+1| \qquad x(0) = -1$$

$$-1 = c\cdot(0+1) + (0+1)\cdot\ln|0+1|$$

$$-1 = c$$

$$x(t) = -1\cdot(t+1) + (t+1)\cdot\ln|t+1| =$$

$$\boxed{= (t+1)\cdot(\ln|t+1| - 1)\ ;\ t \in (-1; +\infty)}$$

47/9 j) $x' = \frac{-2t}{t^2+1}x + \frac{1}{t^2+1}$ $\quad x(1) = 0 \quad t \in R$

$$\frac{dx}{dt} = \frac{-2t}{t^2+1}x$$

$$\int \frac{1}{x}\,dx = -\int \frac{2t}{t^2+1}\,dt$$

$$\ln|x| = -\ln|t^2+1| + \ln c$$

$$\tilde{x} = \frac{c}{t^2+1} = c\cdot(t^2+1)^{-1}$$

$$\tilde{x}' = c'\cdot(t^2+1)^{-1} + c\cdot 2t\cdot[-(t^2+1)^{-2}]$$

$$c'\cdot(t^2+1)^{-1} \cancel{-2ct\cdot(t^2+1)^{-2}} = \cancel{\frac{-2t}{t^2+1}\cdot c(t^2+1)^{-1}} + \frac{1}{t^2+1}$$

$$c'\cdot(t^2+1)^{-1} = \frac{1}{t^2+1}$$

$$c' = 1$$

$$c = t + k$$

$$\hat{x} = \frac{t}{t^2+1}$$

$$x(t) = \frac{c}{t^2+1} + \frac{t}{t^2+1}\ ;\ -\infty R \qquad x(1) = 0$$

$$0 = \frac{c}{1+1} + \frac{1}{1+1}$$

$$0 = \frac{c}{2} + \frac{1}{2}$$

$$c = -1$$

$$\boxed{x(t) = \frac{t}{t^2+1} - \frac{1}{t^2+1}\ ;\ t \in R}$$

$$\boxed{= \frac{t-1}{t^2+1}}$$

47/10a

$x' - 3x = 0$

$\lambda - 3 = 0$

$\lambda = 3$

$\boxed{x(t) = c \cdot e^{3t} \quad t, c \in \mathbb{R}}$

b)

$2x' + x = 0$

$2\lambda + 1 = 0$

$\lambda = -\frac{1}{2}$

$\boxed{x(t) = c \cdot e^{-\frac{t}{2}} \quad t, c \in \mathbb{R}}$

c)

$x'' + x' - 2x = 0$

$\lambda^2 + \lambda - 2 = 0$

$\lambda_{12} = \frac{-1 \pm 3}{2} = \begin{cases} -2 \\ 1 \end{cases}$

$\boxed{x(t) = c_1 \cdot e^{t} + c_2 \cdot e^{-2t} \quad c_1; c_2; t \in \mathbb{R}}$

d)

$2x'' + x' - x = 0$

$2\lambda^2 + \lambda - 1 = 0$

$D = 1 + 8 = 9$

$\lambda_{12} = \frac{-1 \pm 3}{4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$

$\boxed{x(t) = c_1 \cdot e^{-t} + c_2 \cdot \sqrt{e^{t}} \quad c_1; c_2; t \in \mathbb{R}}$

e)

$x'' + 4x' = 0$

$\lambda^2 + 4\lambda = 0$

$\lambda \cdot (\lambda + 4) = 0$

$\lambda_1 = 0$

$\lambda_2 = -4$

$x(t) = c_1 \cdot e^{0t} + c_2 \cdot e^{-4t} =$

$= \boxed{c_1 + c_2 \cdot e^{-4t} \quad c_1; c_2; t \in \mathbb{R}}$

f)

$x'' = 0$

$\lambda^2 = 0$

! $\lambda_1 = 0$, $\lambda_2 = 0$ $\Rightarrow \{e^{0t}; te^{0t}\}$ – fundament. řešení !

$x(t) = c_1 + c_2 t \quad ; c_1; c_2; t \in \mathbb{R}$

g)

$x'' - 2x' + x = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$D = 4 - 4 = 0$

$\lambda_{12} = \frac{-b}{2a} = \frac{2}{2} = 1$

$\lambda_1 = 1$

$\lambda_2 = 1$ (protože 2. derivace)

$\{e^{t}; te^{t}\}$

$\boxed{x(t) = c_1 e^{t} + c_2 t e^{t} = e^{t} \cdot (c_1 + c_2 t) \quad c_1; c_2; t \in \mathbb{R}}$

47) 10 l)

$x''' - 6x'' + 9x' = 0$

$\lambda^2 - 6\lambda + 9 = 0$

$D = 36 - 36 = 0$

$\lambda_{2,3} = \frac{6}{2} = 3$

$\lambda_1 = 0$

$\lambda_{2,3} = 3$

$x(t) = c_1 + e^{3t} \cdot (c_2 + c_3 t) \quad c_1; c_2; c_3; t \in \mathbb{R}$

m) $x''' + 4x' = 0$

$\lambda^2 + 4 = 0$

$\lambda_{2,3} = \pm 2j$

$\lambda_1 = 0$

$\lambda_2 = 2j$

$\lambda_3 = -2j$

$x(t) = c_1 + c_2 \cdot \cos 2t + c_3 \cdot \sin 2t \quad ; c_1; c_2; c_3; t \in \mathbb{R}$

n) $x''' + 3x'' + 3x' + x = 0$

$\lambda_1 = -1$

$\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$

$(\lambda + 1)^3 = 0$

$x(t) = c_1 + c_2 t + c_3 t^2 \quad c_1; c_2; c_3; t \in \mathbb{R}$

47/104

$x'' + 9x = 0$

$\lambda^2 + 9 = 0$

$\lambda^2 = -9$

$\lambda = \pm 3j$

$e^{(\alpha \pm \beta j)t} = e^{\alpha t}(\cos \alpha t \mp \sin \beta t)$

$\boxed{x(t) = \cdot^{0} (c_1 \cdot \cos 3t + c_2 \sin 3t) \quad c_1; c_2; t \in R}$

i) $x'' + 2x' + 10x = 0$

$\lambda^2 + 2\lambda + 10 = 0$

$D = 4 - 40 = -36$

$\lambda_{1;2} = \frac{-2 \pm j\sqrt{36}}{2} = \begin{cases} -1 + 3j \\ -1 - 3j \end{cases}$

$\boxed{x(t) = e^{-t} \cdot (c_1 \cdot \cos 3t + c_2 \sin 3t); \quad c_1; c_2; t \in R}$

j) $x'' - 6x' + 13x = 0$

$\lambda^2 - 6\lambda + 13 = 0$

$D = 36 - 52 = -16$

$\lambda_{12} = \frac{6 \pm j\sqrt{16}}{2} = \begin{cases} 3 + 2j \\ 3 - 2j \end{cases}$

$\boxed{x(t) = e^{3t} \cdot (c_1 \cdot \cos 2t + c_2 \cdot \sin 2t) \quad c_1; c_2; t \in R}$

k) $3x''' - 5x'' - 2x' = 0$

$3\lambda^3 - 5\lambda^2 - 2\lambda = 0 \quad /:\lambda$

$3\lambda^2 - 5\lambda - 2 = 0$

$D = 25 + 24 = 49$

$\lambda_{2;3} = \frac{5 \pm 7}{6} = \begin{cases} 2 \\ -\frac{1}{3} \end{cases}$

$x_1 = 0$

$\lambda_2 = 2$

$\lambda_3 = -\frac{1}{3}$

$\boxed{x(t) = c_1 + c_2 \cdot e^{2t} + c_3 \cdot e^{-\frac{1}{3}t} \; ; c_1; c_2; c_3; t \in R}$

úloha 47/11

$x'' - 2x' = h(t)$

$\lambda^2 - 2\lambda = 0 \Rightarrow \lambda_1 = 0$

$\lambda_2 = 2$

a) $h(t) = 2t^2 - t \Rightarrow (At^2 + Bt + C)\, t^k \cdot e^{\alpha t}$ $\Rightarrow \alpha = 0$

$\lambda_1 = 0 \Rightarrow k = 1$

Obecné partikl. řešení je $\underline{\hat{x}(t) = (At^2 + Bt + C) \cdot t}$

b) $h(t) = (t+1)\, e^{-t} \rightarrow (At + B)\, t^k \cdot e^{\alpha t} \rightarrow \alpha = -1$

$k = 0$

Obecné part. řešení je $\underline{\hat{x}(t) = (At + B) \cdot e^{-t}}$

c) $h(t) = (3t - 2)\, e^{2t}$; $(At + B)\, e^{\alpha t} \cdot t^k \rightarrow \alpha = 2$

$\lambda_2 = 2 \Rightarrow k = 1$

$\alpha = 2$

Řešení je $\underline{\hat{x}(t) = t \cdot (At + B)\, e^{2t}}$

d) $h(t) = t \cdot \cos 2t$; $(At + B) \cos 2t + (At + B) \sin 2t$

Řešení je $\underline{\hat{x}(t) = (At + B) \cdot \cos 2t + (At + B) \cdot \sin 2t}$

e) $b(t) = 2e^{2t} \sin t$; $\alpha = 2$, $\lambda_2 = 2 \Rightarrow k = 1$

$m = 1$

Řešení je $\underline{Ae^{2t}\cos t + Be^{2t}\cdot\sin t}$

f) $b(t) = 3e^{t} + 4e^{2t}$

$b_1(t) = 3e^{t} \to \alpha_1 = 1$; $k = 0$

$b_2(t) = 4e^{2t} \Rightarrow \alpha_2 = 2$; $\lambda_2 = 2$; $k = 1$

$$\hat{x}(t) = Ae^{t} + Bt\, e^{2t}$$

47/12 $x'' + 6x' + 9x = b(t)$

$\lambda^2 + 6\lambda + 9 = 0$

$D = 36 - 4.1.9 = 0$

$\lambda_{12} = \frac{-6}{2} = -3$ (! dvojnásobný)

a) $b(t) = (2t-1)e^{3t}$; $(At+B)t^k e^{\alpha t}$; $\alpha = 3 \neq \lambda$, $k = 0$

$$\hat{x}(t) = (At+B)\cdot e^{3t}$$

b) $b(t) = (t+2)e^{-3t}$; $At+B$; $\alpha = -3$, $\lambda_{12} = -3 \Rightarrow k = 2$

$$\hat{x}(t) = t^2(At+B)e^{-3t}$$

c) $b(t) = t\cdot e^{-3t}\cdot \sin t$; $\alpha = -3$

$$\hat{x}(t) = e^{-3t}(At+B)\cdot\cos t + e^{-3t}(Ct+D)\cdot\sin t$$

47/13 $x'' + 4x = b(t)$

$\lambda^2 + 4 = 0$

$\lambda_{12} = \pm 2j$

a) $b(t) = 2t - 1$; $\alpha = 0$; $At + B$

$k = 0$

$$\boxed{\hat{x}(t) = At + B}$$

b) $b(t) = t^2 e^{2t}$; $(At^2 + Bt + C)$ $\alpha = 2 \neq \lambda$

$k = 0$

$$\boxed{\hat{x}(t) = (At^2 + Bt + C)e^{2t}}$$

c) $b(t) = t \cdot \cos 2t$ $(At + B) \cdot \cos 2t + (Ct + D) \cdot \sin 2t$

$\beta_{1} = 2j = \lambda_1 \Rightarrow k = 1$ $\alpha = 0$

$$\boxed{\hat{x}(t) = t(At + B) \cdot \cos 2t + t(Ct + D) \sin 2t}$$

47/14 $x'' + 2x' + 5x = b(t)$

$\lambda^2 + 2\lambda + 5 = 0$

$D = 4 - 20 = -16$

$\lambda_{12} = \frac{-2 \pm j4}{4} \rightarrow -\frac{1}{2} + j$, $-\frac{1}{2} - j$

$= -1 \pm 2j = e^{-t}\cos 2t$, $e^{-t}\sin 2t$

a) $b(t) = (t^2 + 1)e^{-t}$; $\alpha = -1 \neq \lambda$

$n = 2$

$$\boxed{\hat{x}(t) = (At^2 + Bt + C) \cdot e^{-t}}$$

b) $b(t) = 2t \sin 2t$; $\alpha = 0$

$n = 1$

$$\boxed{\hat{x}(t) = (At + B) \cdot \cos 2t + (Ct + D) \cdot \sin 2t}$$

c) $b(t) = 3e^{-t} \cos 2t$; $\alpha = -1$ $\Rightarrow \{-1 + 2j\} = \lambda_1 \Rightarrow k = 1$

$n = 0$

$$\boxed{\hat{x}(t) = te^{-t} A \cdot \cos 2t + te^{-t} B \cdot \sin 2t}$$

48/15

a) $x'' + 2x' - 3x = 0$ $\quad x(0) = 3 \quad x'(0) = -1$

$\lambda^2 + 2\lambda - 3 = 0$

$D = 4 + 12 = 16$

$\lambda_{12} = \frac{-2 \pm 4}{2} \to 1, \ -3$

$\hat{x}(t) = c_1 e^t + c_2 e^{-3t}$

$\hat{x}'(t) = c_1' e^t + c_1 e^t + c_2' e^{-3t} - 3c_2 e^{-3t} = c_1 e^t - 3c_2 e^{-3t}$

$3 = c_1 e^0 + c_2 e^{-3 \cdot 0} \Rightarrow 3 = c_1 + c_2 \Rightarrow c_1 = 3 - c_2$

$-1 = c_1 e^0 - 3c_2 e^{-3 \cdot 0} \Rightarrow -1 = c_1 - 3c_2$

$-1 = 3 - c_2 - 3c_2$

$-4 = -4c_2$

$\boxed{c_2 = 1}$

$c_1 = 3 - 1$

$\boxed{c_1 = 2}$

$\boxed{x(t) = 2e^t + e^{-3t}; t \in \mathbb{R}}$

---

b) $x'' + 5x' = 0$ $\quad x(0) = 0$, $\quad x'(0) = 5$

$\lambda^2 + 5\lambda = 0$

$\lambda \cdot (\lambda + 5) = 0$

$\lambda_1 = 0$

$\lambda_2 = -5$

$\hat{x}(t) = c_1 \cdot + c_2 e^{-5t}$

$\hat{x}'(t) = c_1' + c_2' e^{-5t} + c_2(-5) e^{-5t} = -5c_2 e^{-5t}$

$0 = c_1 + c_2 e^{-5 \cdot 0} \Rightarrow 0 = c_1 + c_2 \Rightarrow c_1 = 1$

$5 = -5c_2 e^{-5 \cdot 0} \Rightarrow 5 = -5c_2 \to c_2 = -1$

$\boxed{\hat{x}(t) = 1 - e^{-5t}}; t \in \mathbb{R}$

48/15c

$$x'' + 4x' + 4x = 0 \qquad x(0) = 2$$
$$x'(0) = -5$$

$$\lambda^2 + 4\lambda + 4 = 0$$

$$D = 16 - 16 = 0$$

$$\lambda_{12} = \frac{-4}{2} = -2$$

$$x(t) = c_1 \cdot e^{-2t} + t \cdot c_2 e^{-2t} = e^{-2t}(c_1 + tc_2) \qquad \{e^{-2t};\ t \cdot e^{-2t}\}$$

$$x'(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t} = e^{-2t} \cdot (-2c_1 + c_2 - 2c_2 t)$$

$$2 = 1 . (c_1) \quad \Rightarrow \quad \underline{c_1 = 2}$$

$$-5 = 1 . (-2c_1 + c_2) \quad -2c_1 + c_2 = -5 \quad \rightarrow \quad -4 + c_2 = -5$$
$$\underline{c_2 = -1}$$

$$\boxed{x(t) = e^{-2t} \cdot (2 - t) \quad ; \quad t \in R}$$

48/15

d)

$$x'' + 2x' + 5x = 0 \qquad x(0) = 0 \; ; \; x'(0) = 6$$

$$\lambda^2 + 2\lambda + 5 = 0 \qquad D = 4 - 20 = -16$$

$$\lambda_{12} = \frac{-2 \pm 4j}{2} = -1 \pm 2j$$

$$\{ e^{-t} \cdot \cos 2t \; ; \; e^{-t} \cdot \sin 2t \}$$

$$\hat{x}(t) = c_1 \cdot e^{-t} \cdot \cos 2t + c_2 \cdot e^{-t} \cdot \sin 2t$$

$$= e^{-t} \cdot (c_1 \cdot \cos 2t + c_2 \cdot \sin 2t)$$

$$\hat{x}'(t) = -e^{-t} \cdot (c_1 \cdot \cos 2t + c_2 \cdot \sin 2t) +$$

$$+ e^{-t} \left[(\cancel{c_1' \cos 2t} - c_1 \cdot 2 \sin 2t) + (\cancel{c_2' \cdot \sin 2t} + 2c_2 \cdot \cos 2t) \right] .$$

$$= -e^{-t} \cdot (c_1 \cdot \cos 2t + c_2 \cdot \sin 2t + 2c_1 \sin 2t - 2c_2 \cdot \cos 2t)$$

$$0 = 1 \cdot (c_1 \cdot 1 + c_2 \cdot 0) \Rightarrow 0 = c_1$$

$$6 = -1 \cdot (c_1 \cdot 1 + \cancel{c_2 \cdot 0} + 2c_1 \cdot 0 - 2c_2 \cdot 1) \Rightarrow 6 = -c_1 + 2c_2$$

$$c_2 = 3$$

$$\boxed{\hat{x}(t) = 3e^{-t} \cdot \sin 2t \; ; \; t \in R}$$

48/15 e)

$$x'' + 3x' + 2x = 6e^{t} \qquad \alpha = 1 \quad m = 0 \Rightarrow A \cdot e^{t}$$

$$\lambda^2 + 3\lambda + 2 = 0 \qquad x(0) = 3 \qquad x'(0) = 0$$

$$D = 9 - 8 = 1$$

$$\lambda_{12} = \frac{-3 \pm 1}{2} \to -2, \; -1$$

$$\{ e^{-2t} \; ; \; e^{-t} \}$$

$$\hat{x}(t) = c_1 \cdot e^{-2t} + c_2 \cdot e^{-t} + w(t)$$

$$w(t) = A \cdot e^{t}$$

$$w'(t) = \cancel{A' \cdot e^{t}} + Ae^{t}$$

$$w''(t) = Ae^{t}$$

$$\Downarrow$$

$$Ae^{t} + 3Ae^{t} + 2Ae^{t} = 6e^{t}$$

$$6A = 6$$

$$A = 1$$

$$3 = c_1 + c_2 + 1 \Rightarrow c_2 = 2 - c_1$$

$$0 = -2c_1 - c_2 + 1 \quad (c_2 = 3)$$

$$0 = -2c_1 - (2 - c_1) + 1$$

$$0 = -c_1 - 1$$

$$\boxed{c_1 = -1}$$

$$\hat{x}(t) = c_1 \cdot e^{-2t} + c_2 e^{-t} + e^{t}$$

$$x'(t) = (\cancel{c_1' e^{-2t}} - 2c_1 e^{-2t}) + (\cancel{c_2' e^{-t}} - c_2 e^{-t}) + e^{t}$$

$$\boxed{x(t) = 3e^{-t} - e^{-2t} + e^{t} \; ; \; t \in R}$$

4P/1Γ f)

$$x'' + 2x' + x = 2\sin t \qquad x(0)=0$$
$$x'(0)=0$$
$$\lambda^2 + 2\lambda + 1 = 0$$

$$\Delta = 4-4=0$$
$$\lambda_{12} = \frac{-2}{2} = -1 \ ; k=1$$
$$\{e^{-t}; te^{-t}\}$$

$$x(t) = c_1 \cdot e^{-t} + c_2 \cdot t \cdot e^{-t} + w(t)$$

$$w(t) = A\cdot \cos t + B \cdot \sin t$$
$$w'(t) = B\cdot \cos t - A \cdot \sin t$$
$$w''(t) = -(A\cdot \cos t + B\cdot \sin t)$$

$$w''(t) + 2\cdot w'(t) + w(t) = 2\sin t$$
$$\cancel{-A\cos t} - \cancel{B\sin t} + 2B\cos t - 2A\sin t + \cancel{A\cos t} + \cancel{B\sin t} = 2\sin t$$
$$\cos t\,(-A + 2B + A) + \sin t\,(-B - 2A + B) = 2\sin t$$
$$2B\cos t - 2A\sin t = 2\sin t$$
$$\cos t: \quad 2B = 0 \Rightarrow B = 0$$
$$\sin t: \quad -2A = 2 \Rightarrow \boxed{A = -1}$$

$$x(t) = c_1 \cdot e^{-t} + c_2 t e^{-t} - \cos t = e^{-t}\cdot(c_1 + c_2 t) - \cos t$$
$$x'(t) = (\cancel{c_1' e^{-t}} - c_1 e^{-t}) + (\cancel{c_2' t\cdot e^{-t}} + c_2 e^{-t} - c_2\cdot t\cdot e^{-t}) + \sin t$$

$$0 = c_1 - 1 \Rightarrow c_1 = 1$$
$$0 = -c_1 + c_2 \Rightarrow c_2 = 1$$

$$\boxed{x(t) = e^{-t}(1+t) - \cos t \ ; t \in \mathbb{R}}$$

48/15 g

$x'' + 4x = 3\cos t \qquad x(0) = 4 \qquad x \in \mathbb{R}$

$x'(0) = 2$

$\lambda^2 + 4 = 0$

$\lambda_{12} = \pm 2j \qquad \{c_1 \cos 2t ; c_2 \cdot \sin 2t\}$

$\hat{x}(t) = c_1 \cdot \cos 2t + c_2 \cdot \sin 2t + w(t)$

$w(t) = A \cos t + B \cdot \sin t \qquad \Rightarrow \cancel{w(t) \cos} \; w(t) = \cos t$

$w'(t) = B \cdot \cos t - A \cdot \sin t$

$w''(t) = -A \cdot \cos t - B \cdot \sin t$

$w'' + 4w = 3\cos t$

$\cancel{-A \cdot \cos t} - B \cdot \sin t + \cancel{4A \cos t} + 4B \sin t = 3 \cos t$

$\cos t \cdot (-A + 4A) + \sin t (-B + 4B) = 3 \cos t$

$3A \cos t + 3B \sin t = 3 \cos t$

$\cos t : \quad 3A = 3 \qquad \Rightarrow A = 1$

$\sin t : \quad 3B = 0 \qquad \Rightarrow B = 0$

$\hat{x}(t) = c_1 \cdot \cos 2t + c_2 \cdot \sin 2t + \cos t$

$x'(t) = \cancel{c_1' \cos t} + 2c_1 (-\sin 2t) + \cancel{c_2' \sin 2t} + 2c_2 \cdot \cos 2t - \sin t$

$= -2c_1 \cdot \sin 2t + 2c_2 \cdot \cos 2t - \sin t$

$4 = c_1 + 1 \qquad \Rightarrow c_1 = 3$

$2 = 2c_2 \qquad \Rightarrow c_2 = 1$

$\underline{x(t) = 3\cos 2t + \sin 2t + \cos t \quad ; t \in \mathbb{R}}$

4P/15h

$$x'' + 2x' + 5x = 3e^{-t}\sin t \qquad x(0) = 3,\quad x'(0) = -2$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$D = 4 - 20 = -16$$

$$\lambda_{12} = \frac{-2 \pm 4j}{2} = \begin{cases} \to -1 + 2j \\ \to -1 - 2j \end{cases}$$

$$\{e^{-t}\cos 2t,\ e^{-t}\cdot\sin 2t\}$$

$$x(t) = c_1\cdot e^{-t}\cdot\cos 2t + c_2\cdot e^{-t}\cdot\sin 2t + w(t) = e^{-t}\cdot(c_1\cdot\cos 2t + c_2\sin 2t) + w(t)$$

$$w(t) = A\cdot e^{-t}\cdot\cos t + B\cdot e^{-t}\cdot\sin t = e^{-t}\cdot(A\cdot\cos t + B\cdot\sin t)$$

$$w'(t) = -e^{-t}(A\cos t + B\cdot\sin t) + e^{-t}(-A\cdot\sin t + B\cdot\cos t)$$

$$= e^{-t}\cdot(\sin t\cdot(-A - B) + \cos t\cdot(B - A))$$

$$w''(t) = e^{-t}\cdot(\cos t\cdot(-A - B) - \sin t\cdot(B - A)) - e^{-t}\cdot(\sin(-A - B) + \cos t\cdot(B - A))$$

$$= e^{-t}\cdot(\cos t\cdot(-A - B + A - B) - \sin t\cdot(B - A - A - B)) =$$

$$= e^{-t}\cdot(\cos t\cdot(-2B) - \sin t\cdot(-2A))$$

$$e^{-t}\cdot\left[\cancel{\cos t\cdot(-2B)} + \cancel{\sin t\cdot(2A)} + \cancel{\sin t\cdot(-2A - 2B)} + \cancel{\cos t\cdot(2B - 2A)} + \cancel{5A\cos t} + 5B\sin t\right) =$$

$$\cos t\cdot(-2B + 2B - 2A + 5A) + \sin t(2A - 2A - 2B + 5B) = 3\sin t \qquad = 3\cancel{e^{-t}}\sin t$$

$$\cos t:\ +3A = 0 \quad\Rightarrow A = 0$$

$$\sin t:\ 3B = 3 \quad\Rightarrow B = 1 \quad\Rightarrow w(t) = e^{-t}\cdot\sin t$$

$$x(t) = e^{-t}\cdot(c_1\cdot\cos 2t + c_2\cdot\sin 2t) + e^{-t}\sin t = e^{-t}\cdot(c_1\cdot\cos 2t + c_2\cdot\sin 2t + \sin t)$$

$$x'(t) = -e^{-t}\cdot(c_1\cdot\cos 2t + c_2\cdot\sin 2t + \sin t) + e^{-t}\cdot(c_1\cdot 2\cdot(-\sin 2t) + c_2\cdot 2\cos 2t + \cos t)$$

$$3 = 1\cdot(c_1) \Rightarrow c_1 = 3$$

$$-2 = -1\cdot(c_1) + 1\cdot(c_2 + 1) \Rightarrow -2 = -c_1 + c_2 + 1$$

$$-2 = -3 + c_2 + 1 \Rightarrow c_2 = 0$$

$$\boxed{x(t) = e^{-t}\cdot(3\cdot\cos 2t + \sin t);\ t \in R}$$

48/15:

$$x'' + 3x' + 2x = 2t\,e^{-t} \qquad ?\ x(0)=0\ ?;\ x'(0)=0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$a = -1;\ b = 1$

$D = 9 - 8 = 1$

$$\lambda_{1,2} = \frac{-3 \pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$$\{e^{-t}; e^{-2t}\}$$

$$x(t) = c_1 \cdot e^{-t} + c_2 \cdot e^{-2t} + w(t)$$

$$w(t) = (At + B) \cdot t \cdot e^{-t} = (At^2 + Bt)\,e^{-t}$$

$$w'(t) = (2At + B)e^{-t} - e^{-t}\cdot(At^2 + Bt) = e^{-t}\cdot(-At^2 + 2At - Bt + B)$$

$$w''(t) = e^{-t}\cdot(-2At + 2A - B) - e^{-t}(-At^2 + 2At - Bt + B) =$$

$$= e^{-t}\cdot(At^2 - 2At + Bt - B - 2At + 2A - B) = e^{-t}\cdot(At^2 - 4At + Bt + 2A - 2B)$$

$$e^{-t}\left[(\cancel{At^2} - \cancel{4At} + \cancel{Bt} + 2A - 2B) + (\cancel{-3At^2} + 6At - 3Bt + 3B) + (\cancel{2At^2} + 2Bt)\right] = 2t\,e^{-t}$$

$$0At^2 + 2At + 0Bt + 2A + B = 2t$$

t^2: $0 = 0$

t^1: $2A = 2 \Rightarrow A = 1$

t^0: $2A + B = 0 \Rightarrow B = -2$ $\qquad \Rightarrow w(t) = (t^2 - 2t)\,e^{-t}$

$$x(t) = c_1 \cdot e^{-t} + c_2 \cdot e^{-2t} + (t^2 - 2t)\,e^{-t} = e^{-t}\cdot(c_1 + c_2 e^{-t} + t^2 - 2t)$$

$$x'(t) = e^{-t}\cdot(-c_2 e^{-t} + 2t - 2) - e^{-t}\cdot(c_1 + c_2 e^{-t} + t^2 - 2t) =$$

$$= e^{-t}\cdot(\cancel{-c_2 e^{-t}} + \cancel{2t} - 2 - c_1 - c_2 e^{-t} - \cancel{t^2} + \cancel{2t}) = e^{-t}\cdot(-2c_2 e^{-t} + 4t - t^2 - c_1 - 2)$$

$x(0) = 0 \Rightarrow 0 = 1\cdot(c_1 + c_2 + 0 + 0) \Rightarrow c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$

$x'(0) = 0 \Rightarrow 0 = 1\cdot(-2c_2 - c_1 - 2) \Rightarrow 2c_2 + c_1 = -2 \qquad c_1 = 2$

$2c_2 - c_2 = -2$

$c_2 = -2$

$$\boxed{x(t) = e^{-t}\cdot(t^2 - 2t - 2 - 2e^{-t}) = e^{-t}\cdot(t^2 - 2t + 2) - 2e^{-2t};\ t \in \mathbb{R}}$$

48/15 j $\quad x'' + x = \cos t \quad x(0) = 1 \quad x'(0) = 0$

$\lambda^2 + 1 = 0 \quad \Rightarrow \lambda_{12} = \pm j$

$\{\cos t ; \sin t\}$

$k = 1$

$x(t) = c_1 \cdot \cos t + c_2 \cdot \sin t + w(t)$

$w(t) = At \cos t + Bt \cdot \sin t$

$w'(t) = A\cos t - At \sin t + B \sin t + Bt \cos t = \cos t(A + Bt) + \sin t(B - At)$

$w''(t) = -\sin(A + Bt) + \cos t\, B + \cos(B - At) + \sin t \cdot (-A)$

$= \sin t(-2A - Bt) + \cos t(2B - At)$

$\cos t(2B \cancel{- At} \cancel{+ At}) + \sin t(-2A \cancel{- Bt} \cancel{+ Bt}) = \cos t$

$2B\cos t - 2A \sin t = \cos t$

$\cos t : 2B = 1 \quad \Rightarrow B = \frac{1}{2}$

$\sin t \cdot -2A = 0 \quad \Rightarrow A = 0 \quad \Rightarrow w(t) = \frac{1}{2} t \cdot \sin t$

$x(t) = c_1 \cdot \cos t + c_2 \cdot \sin t + \frac{1}{2} t \cdot \sin t$

$x'(t) = -c_1 \cdot \sin t + c_2 \cdot \cos t + \frac{1}{2} \sin t + \frac{1}{2} t \cdot \cos t$

$x(0) = 1 \quad 1 = c_1$

$x'(0) = 0 \quad 0 = c_2$

$x(t) = \cos t + \frac{1}{2} t \cdot \sin t =$

$= \underline{\left(2\cos t + t \cdot \sin t ;\ t \in \mathbb{R} \right)}$

49/16 a) $x'' + x = \frac{1}{\sin t}$ $\quad x\left(\frac{\pi}{2}\right) = 0$ $\quad t \neq \pi k \; ; k \in \mathbb{Z}$

$x'\left(\frac{\pi}{2}\right) = 0$

$\lambda^2 + 1 = 0$

$\lambda_{12} = \pm 1 j \qquad \{\cos t ; \sin t\}$

$x'(t) = c_1 \cdot \cos t + c_2 \cdot \sin t$

$$\boxed{\begin{array}{l} c_1' \cdot \cos t + c_2' \cdot \sin t = 0 \\ -c_1' \cdot \sin t + c_2' \cdot \cos t = \frac{1}{\sin t} \end{array}} \Rightarrow c_1' = \frac{-c_2 \cdot \sin t}{\cos t}$$

$$\frac{c_2' \cdot \sin^2 t}{\cos t} + c_2' \cdot \cos t = \frac{1}{\sin t} \qquad c_1' = -\cancel{\frac{\cos t}{\sin t}} \cdot \cancel{\frac{\sin t}{\cos t}}$$

$$c_1' = -1$$

$$c_2' \cdot \left(\frac{\sin^2 t + \cos^2 t}{\cos t}\right) = \frac{1}{\sin t}$$

$$c_2' = \frac{\cos t}{\sin t}$$

$c_1 = \int c_1' dt \Rightarrow c_1 = -t + k_1$

$c_2 = \int c_2' dt \Rightarrow c_2 = \ln(\sin t) + k_2$

$x(t) = (k_1 - t) \cdot \cos t + (k_2 + \ln(\sin t)) \cdot \sin t$

$x'(t) = (-1) \cdot \cos t + (k_1 - t) \cdot (-\sin t) + \frac{1}{\sin t} \cdot \sin t + (k_2 + \ln(\sin t)) \cdot \cos t$

$x\left(\frac{\pi}{2}\right) = 0 \qquad 0 = \left(k_1 - \frac{\pi}{2}\right) \cdot 0 + (k_2 + 0 \cdot \sin t)$

$x'\left(\frac{\pi}{2}\right) = 0 \qquad 0 = (-1) \cdot 0 + \left(k_1 - \frac{\pi}{2}\right) \cdot (-1) + 1 \cdot 1 + (k_2 + 0) \cdot 0$

$0 = k_2 \qquad k_2 = 0$

$0 = \frac{\pi}{2} - k_1 + k_2 \Rightarrow k_1 = \frac{\pi}{2}$

$$x(t) = \left(\frac{\pi}{2} - t\right) \cdot \cos t + \ln(\sin t) \cdot \sin t \; ; \; t \in (0 ; \pi)$$

4P/16 b

$$x'' + x = \frac{1}{\cos^3 t} \qquad x(0) = 1 \ ; \ x'(0) = 1$$

$$\lambda^2 + 1 = 0 \quad \Rightarrow \quad \lambda_{12} = \pm j \quad \Rightarrow \quad \{\cos t, \sin t\}$$

$$\tilde{x}(t) = c_1 \cdot \cos t + c_2 \cdot \sin t$$

$$\hat{x}(t) = \quad c_1' \cdot \cos t + c_2' \cdot \sin t = 0 \qquad \Rightarrow c_1' = -c_2 \cdot \frac{\sin t}{\cos t}$$

$$c_1' \cdot (-\sin t) + c_2' \cdot \cos t = \frac{1}{\cos^3 t}$$

$$\boxed{c_1' = -\frac{\sin t}{\cos^3 t}}$$

$$-c_2' \cdot \frac{\sin t \cdot (-\sin t)}{\cos t} + c_2' \cdot \cos t = \frac{1}{\cos^3 t}$$

$$c_2' \cdot \frac{\sin^2 t}{\cos t} + c_2' \cdot \frac{\cos^2 t}{\cos t} = \frac{1}{\cos^3 t}$$

$$c_2' \cdot \left(\frac{1}{\cos t}\right) = \frac{1}{\cos^3 t} \rightarrow \boxed{c_2' = \frac{1}{\cos^2 t}}$$

$$c_1 = \int c_1' dt = -\int \frac{\sin t}{\cos^3 t} dt = \left| \begin{array}{l} u = \cos t \\ du = (-\sin t) dt \end{array} \right| = \int \frac{1}{u^3} du =$$

$$= -\frac{1}{u^2} = \boxed{-\frac{1}{\cos^2 t}}$$

$$c_2 = \int c_2' dt = \int \frac{1}{\cos^2 t} dt = \left| \begin{array}{l} u = tg\, t \\ \cos^2 t = \dfrac{1}{1 + tg^2 x} \\ du = \dfrac{1}{\cos^2 t} dt \end{array} \right| = \int 1\, du = u = \boxed{tg\, t}$$

$$\hat{x}(t) = c_1 \cdot \cos t + c_2 \cdot \sin t = -\frac{1}{\cos t} + \frac{\sin^2}{\cos t} = \frac{\sin^2 - 1}{\cos t} = -\frac{\cos^2 t}{\cos t} = \boxed{-\cos t}$$

$$x(t) = \tilde{x}(t) + \hat{x}(t) = c_1 \cdot \cos t + c_2 \cdot \sin t - \cos t =$$

$$= \boxed{(c_1 - 1) \cdot \cos t + c_2 \cdot \sin t}$$

$$x'(t) = (c_1 - 1) \cdot (-\sin t) + c_2 \cdot \cos t$$

$$x(0) = 1 \quad \Rightarrow \quad 1 = (c_1 - 1) \cdot 1 + c_2 \cdot 0 \quad \Rightarrow \quad c_1 = 2$$

$$x'(0) = 1 \qquad 1 = (c_1 - 1) \cdot (-0) \cdot + c_2 \cdot 1 \quad \Rightarrow \quad c_2 = 1$$

$$x(t) = 2\cos t + \sin t - \cos t = \boxed{\cos t + \sin t} \quad ?$$

48/16 b) $x'' + x = \frac{1}{\cos^2 t}$ $\quad x(0) = 0$, $\quad x'(0) = 1$

$\lambda^2 + 1 = 0$

$\lambda_{12} = \pm j \Rightarrow \{\cos t;\ \sin t\}$

$\tilde{x}(t) = c_1 \cdot \cos t + c_2 \cdot \sin t$

$\hat{x}(t)$: $\quad c_1' \cdot \cos t + c_2' \cdot \sin t = 0 \Rightarrow c_1 = -c_2' \cdot \frac{\sin t}{\cos t}$

$-c_1' \cdot (\sin t) + c_2' \cdot \cos t = \frac{1}{\cos^3 t}$

$c_2' \frac{\sin^2}{\cos t} + c_2' \cdot \frac{\cos^2 t}{\cos t} = \frac{1}{\cos^3 t}$ $\qquad c_1' = -\frac{\sin t}{\cos^3 t}$

$c_2' = \frac{1}{\cos^2 t}$

$c_1 = \int -\frac{\sin t}{\cos^3 t} dt \left| \begin{array}{l} \cos t = u \\ -\sin t\,dt = du \end{array} \right| = \int \frac{1}{u^3} du = -\frac{1}{u^2} = \boxed{-\frac{1}{\cos^2 t} + k_1}$

$c_2 = \int \frac{1}{\cos^2 t} dt = \left| \begin{array}{l} x = tg\, t \\ dx = \frac{1}{\cos^2 x} \\ \cos^2 t = \frac{1}{1 + tg\, x} \end{array} \right| = \int 1\, dx = x = tg\, t$

$\boxed{= \frac{\sin t}{\cos t} + k_2}$

$x^*(t) = \left(k_1 - \frac{1}{\cos^2 t}\right) \cos t + \left(k_2 + tg\, t\right) \cdot \sin t$

$x'(t) = \left[\left(-\frac{\sin t}{\cos^3 t}\right) \cos t + \left(k_1 - \frac{1}{\cos^2 t}\right) \cdot (-\sin t)\right] +$

$+ \left[\frac{1}{\cos^2 t} \cdot \sin t + \left(k_2 + tg\, t\right) \cdot \cos t\right]$

$1 = (k_1 - 1) \Rightarrow k_1 = 2 \qquad k_2 = 1$

$1 = \left[(0) \cdot 1 + \left(k_1 - \frac{1}{1}\right) \cdot 0\right] + \left[\frac{0}{1} + (k_2 + 0) \cdot 1\right]$

$x(t) = 2\cos t - \frac{1}{\cos t} + \sin t + \frac{\sin^2 t}{\cos t} =$

$= \underline{\cos t + \sin t}$?

4P/16c

$x'' - 2x' + x = \frac{e^t}{t}$ $\quad x(1)=0\ ;\ x'(1)=e\ ;\ t \neq 0 \Rightarrow t \in (-\infty;0) \cup \underline{(0;+\infty)}$

$\lambda^2 - 2\lambda + 1 = 0$ $\quad D = 4 - 4\cdot 1 = 0$

$\lambda = 1$

$\{e^t; t\cdot e^t\}$

$x(t) = c_1 \cdot e^t + t \cdot c_2 e^t$

$$\begin{cases} c_1' \cdot e^t + c_2' t \cdot e^t = 0 & \Rightarrow c_1' = -c_2' t \\ c_1' e^t + c_2'(t+1)e^t = \frac{e^t}{t} & c_1' = -\frac{1 t}{t} = -1 \end{cases}$$

$-e^t + c_2' e^t \cdot t + c_2' e^t = \frac{e^t}{t}$

$\cancel{e^t} \cdot (c_2' t + c_2' - 1) = \frac{\cancel{e^t}}{t}$

$c_2' \cdot (t+1) = \frac{1}{t} + 1$

$c_2' \cancel{(t+1)} = \frac{\cancel{1+t}}{t}$

$\boxed{c_2' = t^{-1}}$

$c_1 = \int c_1' dt = -\int 1 dt = -t + k_1$

$c_2 = \int c_2' dt = \int \frac{1}{t} dt = \ln|t| + k_2$

$x(t) = (k_1 - t) \cdot e^t + (\ln|t| + k_2)\, t \cdot e^t$

$x'(t) = (-1) \cdot e^t + (k_1 - t)e^t + \left[(\ln|t| + k_2) \cdot e^t + t \cdot (\ln|t| + k_2) e^t + \left(\frac{1}{t}\right) \cdot t \cdot e^t \right]$

$0 = (k_1 - 1) \cdot e + (0 + k_2) \cdot 1 \cdot e$

$e = -e + (k_1 - 1)e + \left[(0 + k_2)e + 1 \cdot (0 + k_2)e + e \right]$

$0 = k_1 e - e + k_2 e \Rightarrow 1 = k_1 + k_2 \Rightarrow k_1 = 1 - k_2 \Rightarrow \boxed{k_1 = 0}$

$\cancel{e} = -e + k_1 e - e + k_2 e + k_2 e + \cancel{e} \Rightarrow 2e = k_1 e + 2k_2 e \Rightarrow 2 = k_1 + 2k_2$

$2 = 1 - k_2 + 2k_2$

$\boxed{k_2 = 1}$

$x(t) = -te^t + t \cdot (\ln|t| + 1) e^t \ ;\ t \neq 0$

$x(t) = e^t \cdot t (\ln|t| + 1 - 1) = \underline{e^t \cdot t \cdot \ln|t| \ ;\ t \in (0;+\infty)}$

40/16 d

$x'' - 2x' + x = \frac{e^t}{t^2+1}$ $\quad x(0)=1\ ;\ x'(0)=2$

$\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = 1 \Rightarrow \{e^t ; te^t\}$

$x(t) = c_1 \cdot e^t + c_2 te^t + w(t)$

$w(t)$: $\quad c_1' \cdot e^t + c_2' \cdot t \cdot e^t = 0 \quad \Rightarrow c_1' = -c_2' \cdot t$

$c_1' \cdot e^t + c_2' \cdot (t+1) \cdot e^t = \frac{e^t}{t^2+1} \qquad c_1' = \frac{-t}{t^2+1}$

$\cancel{-c_2' t \cdot e^t} + \cancel{c_2' \cdot te^t} + c_2' \cancel{e^t} = \frac{\cancel{e^t}}{t^2+1}$

$c_2' = \frac{1}{t^2+1}$

$c_1 = \int \frac{-t}{t^2+1} dt = -\frac{1}{2} \int \frac{2t}{t^2+1} = -\frac{1}{2} \ln|t^2+1| + k_1$

$c_2 = \int \frac{1}{t^2+1} dt = \arctan t + k_2$

$w(t) = (-\ln\sqrt{t^2+1}) \cdot e^t + \arctan(t) \cdot t \cdot e^t$

$x(t) = c_1 e^t + c_2 te^t + e^t \cdot \left(-\frac{1}{2} \ln(t^2+1)\right) + e^t \cdot \arctan(t) \cdot t$ $\quad - \mathbb{R}$

$\boxed{x(t) = e^t \cdot \left(c_1 + c_2 t - \frac{1}{2} \ln(t^2+1) + t \arctan t\right)}$

$x'(t) = e^t \cdot \left(c_1 + c_2 t - \frac{1}{2} \ln(t^2+1) + t \cdot \arctan t\right) +$
$+ e^t \cdot \left(c_2 - \frac{t}{t^2+1} + \arctan t + \frac{t}{t^2+1}\right)$

$x(0) = 1 \qquad 1 = 1 \cdot (c_1$ $\quad \Rightarrow \boxed{c_1 = 1}$

$x'(0) = 2 \qquad 2 = 1 \cdot (c_1 + 0) + 1 \cdot (c_2 + 0) \quad \Rightarrow c_1 + c_2 = 2 \Rightarrow \boxed{c_2 = 1}$

$\boxed{x(t) = e^t \cdot \left(1 + t - \frac{1}{2} \ln(t^2+1) + t \cdot \arctan t\right)} \quad t \in \mathbb{R}$

48/17a

$$x_1' = 2x_1 - x_2 \qquad x_1(0) = 1$$
$$x_2' = 4x_1 - 3x_2 \qquad x_2(0) = -2$$

$$x_2 = 2x_1 - x_1'$$
$$x_2' = 2x_1' - x_1''$$

$$2x_1' - x_1'' = 4x_1 - 3(2x_1 - x_1')$$
$$2x_1' - x_1'' = -2x_1 + 3x_1'$$
$$-x_1'' - x_1' + 2x_1 = 0$$
$$x_1'' + x_1' - 2x_1 = 0$$
$$\lambda^2 + \lambda - 2 = 0$$

$$D = 1 + 8 = 9$$
$$\lambda_{12} = \frac{-1 \pm 3}{2} \to -2, \; 1$$

$$\boxed{x_1(t) = c_1 \cdot e^{-2t} + c_2 \cdot e^{t}}$$
$$x_1'(t) = -2c_1 e^{-2t} + c_2 e^{t}$$

$$x_2(t) = 2 \cdot (c_1 e^{-2t} + c_2 e^{t}) - (-2c_1 e^{-2t} + c_2 e^{t})$$
$$\boxed{x_2(t) = 4c_1 e^{-2t} + c_2 e^{t}}$$

$$1 = c_1 + c_2 \Rightarrow c_2 = 1 - c_1$$
$$-2 = 4c_1 + c_2 \qquad \boxed{c_2 = 2}$$

$$-2 = 4c_1 + 1 - c_1$$
$$-3 = 3c_1$$
$$\boxed{c_1 = -1}$$

$$X(t) = \begin{Bmatrix} e^{-2t} & e^{t} \\ 4e^{-2t} & e^{t} \end{Bmatrix} ; \quad x(t) = \begin{Bmatrix} -e^{-2t} + 2e^{t} \\ -4e^{-2t} + 2e^{t} \end{Bmatrix} \quad t \in \mathbb{R}$$

18/17 b)

$x_1' = 2x_1 - x_2 \qquad x_1(0) = 5$

$x_2' = -2x_1 + x_2 \qquad x_2(0) = 1 \qquad x_2' = -x_1'$!?

$x_2 = 2x_1 - x_1'$

$x_2' = 2x_1' - x_1''$

$-x_1' = 2x_1' - x_1''$

$x_1'' - 3x_1' = 0 \qquad \lambda_1 = 0$

$\lambda^2 - 3\lambda = 0 \qquad \lambda_2 = 3 \qquad \Rightarrow \boxed{x_1(t) = 1c_1 + c_2 e^{3t}}$

$\lambda.(\lambda - 3) = 0 \qquad x_1'(t) = 3c_2 e^{3t}$

$x_2 = 2(c_1 + c_2 e^{3t}) - 3c_2 e^{3t}$

$\boxed{x_{2(t)} = 2c_1 - c_2 e^{3t}}$

$5 = c_1 + c_2 \quad \Rightarrow \boxed{c_2 = 3}$

$1 = 2c_1 - c_2$

$6 = 3c_1 \Rightarrow \boxed{c_1 = 2}$

$$\boxed{X(t) = \begin{Bmatrix} 1 & e^{3t} \\ 2 & -e^{3t} \end{Bmatrix} \; ; \; x(t) = \begin{Bmatrix} 2 + 3e^{3t} \\ 4 - 3e^{3t} \end{Bmatrix} \quad t \in \mathbb{R}}$$

17 c)

$x_1' = x_1 - x_2 \qquad x_1(0) = -1 \qquad x_2 = x_1 - x_1'$

$x_2' = x_1 + 3x_2 \qquad x_2(0) = 0 \qquad x_2' = x_1' - x_1''$

$\cancel{x_1' - x_1''} = x_1 + 3x_1 \cancel{- 3x_1'}$

$-x_1'' + 4x_1' - 4x_1 = 0 \qquad \lambda^2 - 4\lambda + 4 = 0$

$\lambda_{12} = 2 \Rightarrow \{e^{2t}; t \cdot e^{2t}\}$

$x_1(t) = c_1 \cdot e^{2t} + c_2 t e^{2t}$

$x_1'(t) = 2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t} = 2c_1 e^{2t} + e^{2t} \cdot (c_2 + 2c_2 \cdot t)$

$x_2 = \cancel{c_1 e^{2t}} + c_2 \cdot t \cdot e^{2t} - (\cancel{2c_1 e^{2t}} + e^{2t} (c_2 + 2c_2 t))$

$= -c_1 e^{2t} - e^{2t} c_2 - e^{2t} c_2 \cdot t = -e^{2t} (c_1 + c_2 + c_2 t) = \boxed{-e^{2t} (c_1 + c_2 \cdot (t+1))}$

$\boxed{-1 = c_1}$

$0 = -(c_1 + c_2) \Rightarrow \boxed{c_2 = 1}$

$$\boxed{X(t) = \begin{Bmatrix} e^{2t} & t e^{2t} \\ -e^{2t} & -(t+1) e^{2t} \end{Bmatrix} , \; x(t) = \begin{Bmatrix} e^{2t} \cdot (t - 1) \\ e^{2t} \cdot (-t) \end{Bmatrix} \quad t \in \mathbb{R}}$$

48/17 d

$$x_1' = 2x_1 - 3x_2 \qquad x_1(0) = 2$$
$$x_2' = 3x_1 - 4x_2 \qquad x_2(0) = 1$$

$$3x_2 = 2x_1 - x_1'$$
$$x_2 = \frac{2}{3}x_1 - \frac{1}{3}x_1'$$
$$x_2' = \frac{2}{3}x_1' - \frac{1}{3}x_1''$$

$$\frac{2}{3}x_1' - \frac{1}{3}x_1'' = 3x_1 - \frac{8}{3}x_1 + \frac{4}{3}x_1' \quad / \cdot 3$$
$$\cancel{2x_1' - x_1'' = 9x_1 - 8x_1 + 4x_1'}$$
$$-x_1'' - 2x_1' - x_1 = 0 \qquad \lambda^2 + 2\lambda + 1 = 0$$
$$\lambda_{12} = -1 \Rightarrow \{e^{-t}; t \cdot e^{-t}\}$$
$$x_1(t) = c_1 \cdot e^{-t} + c_2 \cdot t \cdot e^{-t}$$
$$x_1'(t) = -c_1 e^{-t} + e^{-t} \cdot c_2 \cdot (1 - t)$$

$$3x_2 = \cancel{2c_1 \cdot e^{-t} + 2c_2 t \cdot e^{-t}} + \cancel{c_1 e^{-t}} + e^{-t} c_2 (\cancel{t} - 1)$$
$$3x_2 = 3c_1 \cdot e^{-t} + 3t \cdot c_2 e^{-t} - e^{-t} \cdot c_2$$
$$x_2(t) = c_1 \cdot e^{-t} + t \cdot c_2 \cdot e^{-t} - \frac{1}{3} e^{-t} \cdot c_2$$

$$2 = c_1$$
$$1 = c_1 - \frac{1}{3}c_2 \Rightarrow \quad 3 = 3c_1 - c_2$$
$$3 = 6 - c_2$$
$$c_2 = +3$$

$$\boxed{\begin{gathered} X(t) = \begin{Bmatrix} e^{-t} & t \cdot e^{-t} \\ e^{-t} & (3t - 1)e^{-t} \end{Bmatrix} \\[2ex] x(t) = \begin{Bmatrix} e^{-t}(2 + 3t) \\ e^{-t}(1 + 3t) \end{Bmatrix} \end{gathered}} \qquad t \in \mathbb{R}$$

48/17 e)

$x_1' = x_1 + x_2 \qquad x_1(0) = 1 \qquad x_2 = x_1' - x_1$

$x_2' = -2x_1 + 3x_2 \qquad x_2(0) = 1 \qquad x_2' = x_1'' - x_1'$

$\cancel{x_1'' - x_1'} = -2x_1 + 3x_1' - 3x_1$

$x_1'' - 4x_1' + 5x_1 = 0$

$\lambda^2 - 4\lambda + 5 = 0 \qquad D = 16 - 20 = 4$

$\lambda_{12} = \dfrac{4 \pm 2j}{2} = \begin{cases} 2+j \\ 2-j \end{cases}$

$x_1(t) = c_1 \cdot e^{2t} \cdot \cos t + c_2 \cdot e^{2t} \cdot \sin t$

$x_1'(t) = c_1 e^{2t}(2\cos t - \sin t) + c_2 e^{2t}(2\sin t + \cos t)$

$\{e^{2t}\cos t \, ; \, e^{2t}\sin t\}$

$x_2 = x_1' - x_1$

$x_2 = c_1 e^{2t}(2\cos t - \sin t - \cos t) + c_2 e^{2t}(2\sin t + \cos t - \sin t)$

$x_2(t) = c_1 e^{2t}(\cos t - \sin t) + c_2 e^{2t}(\sin t + \cos t)$

$1 = c_1$

$1 = c_1 + c_2 \Rightarrow \underline{\underline{c_2 = 0}}$

$$X(t) = \begin{Bmatrix} e^{2t}\cos t & e^{2t}\sin t \\ e^{2t}(\cos t - \sin t) & e^{2t}(\sin t + \cos t) \end{Bmatrix}$$

$$x(t) = \begin{Bmatrix} e^{2t}\cos t \\ e^{2t}(\cos t - \sin t) \end{Bmatrix} \; ; \quad t \in \mathbb{R}$$

48/17 f

$$x_1' = -x_2 \qquad x_1(0) = 1 \qquad x_2 = -x_1'$$
$$x_2' = 2x_1 + 2x_2 \qquad x_2(0) = 1 \qquad x_2' = -x_1''$$

$$-x_1'' = 2x_1 + (-2x_1')$$
$$-x_1'' + 2x_1' - 2x_1 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0 \qquad D = 4 - 8 = -4$$
$$\lambda_{12} = \frac{2 \pm 2j}{2} \Rightarrow 1 \pm j \quad \{e^t \cos t ; e^t \sin t\}$$

$$x_1(t) = c_1 \cdot e^t \cdot \cos t + c_2 e^t \cdot \sin t$$
$$x_1'(t) = (e^t \cdot \cos t - e^t \sin t) c_1 + c_2 \cdot (e^t \cdot \sin t + e^t \cdot \cos t)$$
$$x_2 = -x_1'$$
$$x_2 = c_1 \cdot (e^t \cdot \sin t - e^t \cdot \cos t) - c_2 (e^t \cdot \sin t + e^t \cdot \cos t)$$

$$1 = c_1$$
$$1 = -c_1 - c_2 \Rightarrow c_2 = -2$$

$$X(t) = \begin{Bmatrix} e^t \cos t & e^t \sin t \\ e^t (\sin t - \cos t) & -e^t (\sin t + \cos t) \end{Bmatrix} \quad t \in R$$
$$x(t) = \begin{Bmatrix} e^t \cos t & -2e^t \sin t \\ e^t (\sin t - \cos t) & +2e^t \cdot (\sin t + \cos t) \end{Bmatrix} = \begin{Bmatrix} e^t \cdot (\cos t - 2\sin t) \\ e^t \cdot (3\sin t + \cos t) \end{Bmatrix}$$

48/17 g

$$x_1' = -x_1 + x_2 + e^t \qquad x_1(0) = 0 \qquad x_2 = x_1' + x_1 - e^t$$
$$x_2' = x_1 - x_2 + e^t \qquad x_2(0) = 0 \qquad x_2' = x_1'' + x_1' - e^t$$

$$x_1'' + \cancel{x_1'} - e^t = \cancel{x_1} - x_1' - \cancel{x_1} + e^t + e^t$$
$$x_1'' + 2x_1' = 3e^t$$

$$\lambda^2 + 2\lambda = 0$$
$$\lambda_1 = 0 ; \lambda_2 = -2$$

$$\hat{x}(t) = c_1 + c_2 e^{-2t}$$
$$\tilde{x}(t) = A \cdot e^t$$
$$\tilde{x}'(t) = A e^t$$
$$\tilde{x}''(t) = A e^t$$

$$A e^t + 2A e^t = 3 e^t$$
$$A = 1$$

$$x_1(t) = \underline{c_1 + c_2 e^{-2t} + e^t}$$
$$x_1'(t) = -2c_2 e^{-2t} + e^t$$

$$x_2(t) = -2c_2 e^{-2t} + e^t + c_1 + c_2 e^{-2t} \cancel{+ e^t} \cancel{- e^t}$$
$$\underline{x_2(t) = c_1 - c_2 e^{-2t} + e^t}$$

$$0 = c_1 + c_2 + 1 \qquad \Rightarrow c_2 = 0$$
$$0 = c_1 - c_2 + 1$$

$$0 = 2c_1 + 2 \Rightarrow c_1 = -1$$

$$X(t) = \begin{Bmatrix} 1 & e^{-2t} \\ 1 & -e^{-2t} \end{Bmatrix} ; \quad x(t) = \begin{Bmatrix} e^t - 1 \\ e^t - 1 \end{Bmatrix} ; \quad t \in R$$

48/17h

$x_1' = x_1 - x_2 + 2e^t$ $\qquad x_1(0) = 0 \qquad x_2 = x_1 - x_1' + 2e^t$

$x_2' = -x_1 + x_2 + e^t$ $\qquad x_2(0) = 3 \qquad x_2' = x_1' - x_1'' + 2e^t$

$\cancel{x_1'} - x_1'' + \cancel{2e^t} = (\cancel{-x_1} + \cancel{x_1}) - \cancel{x_1'} + \cancel{2e^t} + e^t$

$-x_1'' + 2x_1' = e^t \qquad \alpha = 1$ $\qquad -\lambda^2 + 2\lambda = 0$

$\lambda_1 = 0\ ;\ \lambda_2 = +2$

$\tilde{x}(t) = c_1 + c_2 e^{+2t}$

$\tilde{x}(t) = Ae^t$ $\qquad -Ae^t + 2Ae^t = e^t$

$\tilde{x}'(t) = Ae^t$ $\qquad -1A + 2A = 1$

$\tilde{x}''(t) = Ae^t$ $\qquad A = 1$

$x_1(t) = c_1 + c_2 \cdot e^{+2t} + e^t$

$x_1'(t) = +2c_2 e^{+2t} + e^t$

$x_2 = c_1 + c_2 e^{+2t} + e^t - (+2c_2 e^{+2t} + e^t) + 2e^t$

$x_2 = c_1 - c_2 e^{+2t} + 2e^t$

$0 = c_1 + c_2 + 1 \quad \Rightarrow c_1 = -1 - c_2$

$3 = c_1 - c_2 + 2 \quad c_1 = 0$

$3 = -1 - c_2 - c_2 + 2$

$2 = -2c_2 \Rightarrow c_2 = -1$

$$X(t) = \begin{Bmatrix} 1 & e^{+2t} \\ 1 & -e^{+2t} \end{Bmatrix}$$

$t \in \mathbb{R}$

$$x(t) = \begin{Bmatrix} -e^{2t} + e^t \\ e^{2t} + 2e^t \end{Bmatrix}$$