78/1a

a) $2t^2 - 3t + 4 \;\hat{=}\; \frac{4}{p^3} - \frac{3}{p^2} + \frac{4}{p} \;;\; p > 0$

b) $e^{2t} + 3e^{-4t} \;\hat{=}\; \frac{1}{p-2} + \frac{3}{p+4} \;;\; p > 2$

c) $\int_0^{+\infty} \sinh t\, e^{-pt}\, dt = \left| \begin{matrix} u' = \sinh t & v = e^{-pt} \\ u = \cosh t & v' = -pe^{-pt} \end{matrix} \right| = \cosh t\, e^{-pt} + p \int \cosh t \cdot e^{-pt}\, dt =$

$= \left| \begin{matrix} u' = \cosh t & v = e^{-pt} \\ u = \sinh t & v' = -pe^{-pt} \end{matrix} \right| = \cosh t \cdot e^{-pt} + p\left[\sinh t \cdot e^{-pt} + p \int \sinh t \cdot e^{-pt}\, dt\right] =$

$= \cosh t \cdot e^{-pt} + p \cdot \sinh t \cdot e^{-pt} + p^2 \int \sinh t \cdot e^{-pt}\, dt$

$\int \sinh t\, e^{-pt}\, dt - p^2 \int \sinh t\, e^{-pt}\, dt = e^{-pt} \cdot (\cosh t + p \cdot \sinh t)$

$\int \sinh t\, e^{-pt}\, dt \cdot (1 - p^2) = e^{-pt} \cdot (\cosh t + p \cdot \sinh t)$

$\int \sinh t\, e^{-pt}\, dt = \frac{e^{-pt} (\cosh t + p \cdot \sinh t)}{1 - p^2}$

$\lim_{t \to \infty} e^{-pt} (\cosh t + p \cdot \sinh t) \overset{LH}{=} 0 \cdot (\text{omez}) = 0$

$\lim_{t \to 0} e^{-pt} \cdot (\cosh t + p \cdot \sinh t) = 1 \cdot (1 + p \cdot 0) = 1$

$(1 - p^2)$ – konstanta => netřeba označovat v limitě (se píše ní)

$\int_0^{+\infty} \sinh t\, e^{-pt}\, dt = \frac{1}{1-p^2} \cdot \left[e^{-pt} \cdot (\cosh t + p \cdot \sinh t) \right]_0^{+\infty} = \frac{1}{1-p^2} \cdot (0 - (+1)) =$

$= -\frac{1}{1-p^2} = \boxed{\frac{1}{p^2-1}} \;;\; p > 1$

781 d

$\mathcal{L}\{\cosh(2t)\}$ $\qquad I = \cosh(2t)e^{-pt}$

$$\int_0^{+\infty}\cosh(2t)e^{-pt}\,dt = \left|\begin{matrix} u' = \cosh(2t) & v' = e^{-pt} \\ u = \frac{1}{2}\sinh(2t) & v' = -pe^{-pt}\end{matrix}\right| = \frac{1}{2}\sinh(2t)\cdot e^{-pt} + \frac{p}{2}\int \sinh(2t)e^{-pt}\,dt =$$

$$= \left|\begin{matrix} u' = \sinh(2t) & v = e^{-pt} \\ u = \frac{1}{2}\cosh(2t) & v' = -pe^{-pt}\end{matrix}\right| = \frac{1}{2}\sinh(2t)\cdot e^{-pt} + \frac{p}{2}\left[\frac{1}{2}\cosh(2t)e^{-pt} + \frac{p}{2}\int \cosh(2t)e^{-pt}\,dt\right]$$

$$= \frac{1}{2}\sinh(2t)e^{-pt} + \frac{p}{4}\cosh(2t)e^{-pt} + \frac{p^2}{4}\int\cosh(2t)e^{-pt}\,dt \Rightarrow$$

$$\Rightarrow \quad I - \frac{p^2}{4}I = \frac{e^{-pt}}{2}\left(\sinh(2t) + \frac{p}{2}\cosh(2t)\right)$$

$$I\cdot\left(1 - \frac{p^2}{4}\right) = \frac{e^{-pt}}{2}\cdot\left(\sinh(2t) + \frac{p}{2}(\cosh(2t)\right)$$

$$I = \frac{\frac{e^{-pt}}{2}\cdot\left(\sinh(2t) + \frac{p}{2}\cdot\cosh(2t)\right)}{1 - \frac{p^2}{4}}$$

$$\frac{1}{1 - \frac{p^2}{4}}\lim_{t\to\infty}\frac{e^{-pt}}{2}\left(\sinh(2t) + \frac{p}{2}\cdot\cosh(2t)\right) = \frac{1}{1-\frac{p^2}{4}}\cdot 0.(\text{omez}) = 0$$

$$\frac{1}{1 - \frac{p^2}{4}}\lim_{t\to 0}\frac{e^{-pt}}{2}\left(\sinh(2t) + \frac{p}{2}(\cosh(2t)\right) = \frac{1}{1-\frac{p^2}{4}}\cdot\frac{1}{2}\left(0 + \frac{p}{2}\right) =$$

$$= \frac{1}{\frac{4-p^2}{4}}\cdot\frac{p}{4} = \frac{p}{\frac{4(4-p^2)}{4}} = \frac{p}{4-p^2}$$

$$\int_0^{\infty}\cosh(2t)e^{-pt}\,dt = \frac{\frac{1}{2}}{1 - \frac{p^2}{4}}\cdot\left[e^{-pt}\cdot\sinh(2t) + e^{-pt}\cdot\frac{p}{2}\cdot\cosh(2t)\right]_0^{\infty} =$$

$$= \frac{2}{4-p^2}\cdot\left(0 - \left(0 + \frac{p}{2}\right)\right) = -\frac{p}{4-p^2} = \boxed{\frac{p}{p^2-4}\quad ; p > 2}$$

78/1e

$$3\sin t - 2\cos t \mathrel{\hat=} \frac{3}{p^2+1} - \frac{2p}{p^2+1} = \frac{3-2p}{p^2+1} \; ; \; p>0$$

f) $$4\cos 2t + 3\sin 2t \mathrel{\hat=} \frac{4p}{p^2+4} + \frac{6}{p^2+4} = \frac{4p+6}{p^2+4} \; ; \; p>0$$

g) $$3te^{-t} + 2t^2e^{3t} \mathrel{\hat=} \frac{3}{(p+1)^2} + \frac{4}{(p-3)^3} \; ; \; p>3$$

h) $$t\sin 2t \mathrel{\hat=} -\left(\frac{2}{p^2+4}\right)' = -\frac{4p}{(p^2+4)^2}$$

i) $$t\cdot\cos 3t \mathrel{\hat=} -\left(\frac{p}{p^2+9}\right)' = -\left(\frac{p^2+9-2p^2}{(p^2+9)^2}\right) = \frac{p^2-9}{(p^2+9)^2}$$

j) $$t^2\cdot\sin 3t \mathrel{\hat=} (-1)^2\left(\frac{3}{p^2+9}\right)'' = \left(\frac{-6p}{(p^2+9)^2}\right)' =$$

$$= \left(\frac{-6\cdot(p^2+9)^2 + 6p\cdot[2p\cdot 2\cdot(p^2+9)]}{(p^2+9)^4}\right) = \frac{-6(p^2+9)^{\cancel{2}} + 24p^2\cancel{(p^2+9)}}{(p^2+9)^{\cancel{4}\,3}} =$$

$$= \frac{-6p^2-54+24p^2}{(p^2+9)^3} = \frac{18p^2-54}{(p^2+9)^3} = \boxed{\frac{18(p^2-3)}{(p^2+9)^3} \; ; \; p>0}$$

k) $$t^2\cdot\cos 2t \mathrel{\hat=} (-1)^2\left(\frac{p}{p^2+4}\right)'' = \left(\frac{(p^2+4)-p\cdot 2p}{(p^2+4)^2}\right)' = \left(\frac{4-p^2}{(p^2+4)^2}\right)' =$$

$$= \left(\frac{-2p\cdot(p^2+4)^{\cancel{2}} - [(4-p^2)\cdot 2p\,2\cancel{(p^2+4)}]}{(p^2+4)^{\cancel{4}\,3}}\right) = \frac{-2p\cdot(p^2+4) - 4p\cdot(4-p^2)}{(p^2+4)^3} =$$

$$\frac{-2p\cdot(p^2+4+8-2p^2)}{(p^2+4)^3} = \frac{-2p\cdot(-p^2+12)}{(p^2+4)^3} = \frac{2p^3-24}{(p^2+4)^3} = \boxed{\frac{2(p^3-12)}{(p^2+4)^3}} \quad p>0$$

78/12

$f(t) = 3e^{3t} \cdot \sin 2t \;\hat{=}\; \dfrac{3 \cdot 2}{(p-3)^2+4} = \dfrac{6}{(p-3)^2+4} \quad ; p > 3$

m) $2e^{-t} \cos 3t \;\hat{=}\; \dfrac{2p}{(p+1)^2+9} \quad ; \quad p > 0$

n) $t \cdot e^{2t} \cdot \sin 3t \;\hat{=}\; -\left(\dfrac{3}{(p-2)^2+9}\right)' = -\left(\dfrac{3}{p^2-4p+13}\right)' =$

$= -\left(\dfrac{-3 \cdot (2p-4)}{(p^2-4p+13)^2}\right) = \dfrac{6p-12}{(p^2-4p+13)^2} = \dfrac{6 \cdot (p-2)}{(p^2-4p+13)^2} ; p > 2$

MAPLE OK

o) $f(t) = t \cdot e^{-3t} \cdot \cos 2t = -\left(\dfrac{(p+3)}{(p+3)^2+4}\right)' =$

$= -\left(\dfrac{1 \cdot (p^2+6p+13) - (p+3) \cdot (2p+6)}{((p+3)^2+4)^2}\right) = -\left(\dfrac{\cancel{p^2}+\cancel{6p}+13-\cancel{2p^2}-\cancel{6p}-\cancel{6p}-\cancel{18}}{((p+3)^2+4)^2}\right)$

$= \dfrac{p^2+6p+5+9-9}{(p^2+6p+13)^2} = \dfrac{(p+3)^2-4}{((p+3)^2+4)^2} \quad ; p > 0$

$t^n e^{\alpha t} = \dfrac{n!}{(p-\alpha)^{n+1}}$

$e^{\alpha t} \sin \omega t = \dfrac{\omega}{(p-\alpha)^2+\omega^2} \quad ; \quad e^{\alpha t} \sinh \omega t = \dfrac{\omega}{(p-\alpha)^2-\omega^2}$

$e^{\alpha t} \cos \omega t = \dfrac{p-\alpha}{(p-\alpha)^2+\omega^2} \quad e^{\alpha t} \cosh \omega t = \dfrac{p-\alpha}{(p-\alpha)^2-\omega^2}$

$t^n f(t) = (-1)^n \cdot F^{(n\text{ derivace})}_{(p)}$

$f'(t) = p \cdot F(p) - \lim f(0+)$

$f''(t) = p^2 \cdot F(p) - p \cdot \lim f(0+) - \lim f'(0+)$

$\dfrac{1}{t} f(t) = \int_p^{\infty} F(q)\, dq$

$\int_0^t f(t)\, dt = \dfrac{1}{p} F(p)$

78/2

a) $f(t) = \begin{cases} 1 & t \in \langle 0;2) \\ 0 & t \geq 2 \end{cases}$

$$f(t) = 1.[H(t) - H(t-2)] = H(t) - H(t-2) \;\hat{=}\; \frac{1}{p} - \frac{1}{p}e^{-2p} = \boxed{\frac{1}{p}\cdot\left(1 - e^{-2p}\right)}$$

b) $f(t) = \begin{cases} t & ; t \in \langle 0;2) \\ 0 & t > 2 \end{cases}$

$$f(t) = t.[H(t) - H(t-2)] = t.H(t) - (t-2+2).H(t-2) =$$

$$= t.H(t) - (t-2).H(t-2) - 2.H(t-2) \;\hat{=}\; \frac{1}{p^2} - \frac{1}{p^2}.e^{-2p} - \frac{2}{p}e^{-2p} =$$

$$= \boxed{\frac{1}{p^2} - e^{-2p}.\left(\frac{1}{p^2} + \frac{2}{p}\right)}$$

c) $f(t) = \begin{cases} \sin t & ; t \in \langle 0;\pi) \\ 0 & ; t > \pi \end{cases}$

$$f(t) = \sin t.[H(t) - H(t-\pi)] = \sin t.H(t) - \sin[(t-\pi)+\pi].H(t-\pi) =$$

$$= \sin t.H(t) - [\sin(t-\pi).\cos\pi + \cos(t-\pi).\sin\pi].H(t-\pi) =$$

$$= \sin t.H(t) - [-\sin(t-\pi) + 0]\,H(t-\pi) =$$

$$= \sin t.H(t) + \sin(t-\pi).H(t-\pi) \;\hat{=}\; \frac{1}{p^2+1} + \frac{1}{p^2+1}.e^{-\pi p} =$$

$$= \boxed{\frac{1}{p^2+1}.\left(1 + e^{-\pi p}\right)}$$

d) $f(t) = \begin{cases} t^2 ; & t \in \langle 0;1) \\ 1 ; & t \in \langle 1;+\infty) \end{cases}$

$$f(t) = t^2.[H(t) - H(t-1)] + 1.[H(t-1) - H(t-\infty)] =$$

$$= t^2.H(t) - (t^2-1).H(t-1) - H(t-\infty)$$

78/2

e) $f(t) = \begin{cases} 1 & t \in \langle 0;1) \\ -1 & t \in \langle 1;2) \\ 0 & t \geq 2 \end{cases}$

$$f(t) = 1.[H(t) - H(t-1)] - 1.[H(t-1) - H(t-2)] =$$

$$= H(t) - 2H(t-1) + H(t-2) \,\hat{=}\, \frac{1}{p} - \frac{2}{p}e^{-p} + \frac{1}{p}e^{-2p} =$$

$$= \frac{1}{p}\cdot\left(1 - 2e^{-p} + 1e^{-2p}\right)$$

78/2

f) $f(t) = \begin{cases} t & t \in \langle 0;1) \\ 2-t & t \in \langle 1;2) \\ 0 & t \geq 2 \end{cases}$

$$f(t) = t.[H(t) - H(t-1)] + (2-t).[H(t-1) - H(t-2)] =$$

$$= t.H(t) + (2-2t).H(t-1) - (2-t).H(t-2) =$$

$$= t\cdot H(t) - 2.(t-1).H(t-1) + (t-2).H(t-2) \,\hat{=}$$

$$\hat{=}\, \frac{1}{p^2} - \frac{2}{p^2}e^{-p} + \frac{1}{p^2}e^{-2p} = \frac{1}{p^2}\cdot\left(1 - 2e^{-p} + 1e^{-2p}\right)$$

78/4a

$D = 9 - 8 = 1$

$p_{1,2} = \frac{-3 \pm 1}{2} \nearrow -2, \searrow -1$

2) $\frac{p^2+1}{p^3+3p^2+2p} = \frac{p^2+1}{p\cdot(p^2+3p+2)} = \frac{(p^2+1)}{p\cdot(p+2)\cdot(p+1)} =$

$= \frac{A}{p} + \frac{B}{p+2} + \frac{C}{p+1} =$

$A = \frac{(p^2+1)}{(p+2)\cdot(p+1)}\bigg|_{p=0} = \frac{1}{2}$

$B = \frac{p^2+1}{p(p+1)}\bigg|_{p=-2} = \frac{5}{2}$

$C = \frac{p^2+1}{p\cdot(p+2)}\bigg|_{p=-1} = \frac{2}{-1} = -2$

$= \frac{1}{2}\cdot\frac{1}{p} + \frac{5}{2}\cdot\frac{1}{p+2} - 2\cdot\frac{1}{p+1} \hat{=}$

$\hat{=} \left[\frac{1}{2} + \frac{5}{2}e^{-2t} - 2e^{-t}\right]\cdot H(t)$

x) $\frac{1}{p^3+6p^2+9p} = \frac{1}{p\cdot(p^2+6p+9)} = \frac{A}{p} + \frac{B}{(p+3)^2} + \frac{C}{(p+3)}$

$A = \frac{1}{p^2+6p+9}\bigg|_{p=0} = \boxed{\frac{1}{9}}$

$B = \frac{1}{p}\bigg|_{p=-3} = \boxed{-\frac{1}{3}}$

$1 = A\cdot(p+3)^2 + Bp + Cp\cdot(p+3)$

$0 = A + C \Rightarrow C = -A$

$\boxed{C = -\frac{1}{9}}$

$\frac{1}{9}\cdot\frac{1}{p} - \frac{1}{3}\cdot\frac{1}{(p+3)^2} - \frac{1}{9}\,\frac{1}{p+3} \hat{=} \left[\frac{1}{9} - \left(\frac{1}{3}t\cdot e^{-3t} + \frac{1}{9}e^{-3t}\right)\right]\cdot H(t)$

c) $\frac{1}{(p-1)^2\cdot(p+2)} = \frac{A}{(p-1)^2} + \frac{B}{(p-1)} + \frac{C}{(p+2)}$

$A = \frac{1}{p+2}\bigg|_{p=0} = \frac{1}{2}$

$C = \frac{1}{(p-1)^2}\bigg|_{p=-2} = \frac{1}{9}$

$1 = A\cdot(p+2) + B(p-1)\cdot(p+2) + C(p-1)^2$

$0 = B + C \Rightarrow B = -C$

$B = -\frac{1}{9}$

$\frac{1}{2}\cdot\frac{1}{(p-1)^2} - \frac{1}{9}\cdot\frac{1}{p-1} + \frac{1}{9}\cdot\frac{1}{p+2} \hat{=}$

$\left[\frac{1}{2}t\cdot e^{t} - \frac{1}{9}e^{t} + \frac{1}{9}e^{-2t}\right]H(t) = \left[e^{t}\cdot\left(\frac{1}{2}t - \frac{1}{9}\right) + \frac{1}{9}e^{-2t}\right]H(t)$

78/4

d) $\frac{1}{(p-2)^3} = \frac{A}{(p-2)^3} + \frac{B}{(p-2)^2} + \frac{C}{(p-2)} \mathrel{\hat{=}} \underline{A = 1\ ?}$

$\frac{1}{(p-2)^3} \mathrel{\hat{=}} \left(\frac{1}{2} t^2 e^{2t}\right) \cdot H(t)$

e) $\frac{1}{(p+3)^4} \mathrel{\hat{=}} \left(\frac{1}{6} \cdot t^3 \cdot e^{-3t}\right) \cdot H(t)$

f) $\frac{1}{p^2(p^2+1)} = \frac{A}{p^2} + \frac{B}{p} + \frac{Cx+D}{p^2+1} = \frac{1}{p^2} - \frac{1}{p^2+1} \mathrel{\hat{=}}$

$1 = (\cancel{Ap^2+A}) + (\cancel{Bp^3+Bp}) + (\cancel{Cp^3+Dp^2})$

$p^3: \ 0 = B + C$

$p^2: \ 0 = A + D \quad \Rightarrow D = -A$

$p^1: \ 0 = B \qquad D = -1$

$p^0: \ 1 = A$

$\mathrel{\hat{=}} (t - \sin t) \cdot H(t)$

g) $\frac{1}{p^2+4p+5} = \frac{1}{p^2+4p+4+1} = \frac{1}{(p+2)^2+1} \mathrel{\hat{=}} \qquad D = 16 - 20 = -4\ ?$

$\mathrel{\hat{=}} \left(e^{-2t} \cdot \sin t\right) \cdot H(t)$

h) $\frac{3p+4}{p^2+2p+10} = \frac{3p+4}{(p^2+2p+1)+9} = \qquad D = 4 - 36 = 0$

$= \frac{3(p-1)}{(p+1)^2+3^2} + \frac{4}{(p+1)^2+3^2} \mathrel{\hat{=}} \frac{3p-3+4}{(p+1)^2+3^2} = \quad !!\ H(t)$

$\mathrel{\hat{=}} \frac{3p}{(p+1)^2+3^2} + \frac{1}{(p+1)^2+3^2} = \left[e^{-t} \cdot \left(3\cos 3t + \frac{1}{3} \cdot \sin 3t\right)\right] \cdot H(t)$

78/4

$D = 4 - 20 = -16$

i) $$\frac{4p-3}{p^2-2p+5} = \frac{4p-3}{(p^2-2p+1)+4} = \frac{4(p-1)}{(p-1)^2+2^2} + \frac{-3+4}{(p-1)^2+2^2} \mathrel{\hat=}$$

$$\mathrel{\hat=} \left(4\cdot e^{t}\cos 2t + \frac{1}{2} e^{t}\sin 2t\right)\cdot H(t) = \left[e^{t}\cdot\left(4\cdot\cos 2t + \frac{1}{2}\sin 2t\right)\right] H(t)$$

j) $$\frac{4p+5}{p^2+6p+13} = \frac{4(p+3)-12+5}{(p^2+6p+9)+4} = \frac{4(p+3)}{(p+3)^2+2^2} + \frac{-7}{(p+3)^2+2^2} \mathrel{\hat=}$$

$$\mathrel{\hat=} 4\cdot e^{-3t}\cos 2t - \frac{7}{2} e^{-3t}\sin 2t = \left[e^{-3t}\cdot\left(4\cdot\cos 2t - \frac{7}{2}\sin 2t\right)\right]\cdot H(t)$$

k) $$\frac{p+3}{p^2-4p+20} = \frac{p+3}{(p-2)^2+16} = \frac{(p-2)+5}{(p-2)^2+4^2} \mathrel{\hat=}$$

$$\mathrel{\hat=} \frac{p-2}{(p-2)^2+4^2} + \frac{5}{4}\cdot\frac{4}{(p-2)^2+4^2} \mathrel{\hat=} e^{2t}\cdot\cos 4t + \frac{5}{4}e^{2t}\sin 4t =$$

$$\mathrel{\hat=} \left[e^{2t}\cdot\left(\cos 4t + \frac{5}{4}\sin 4t\right)\right]\cdot H(t)$$

l) $$\frac{2p^3-p^2+1}{p^2-3p}$$ – nemá obraz $Q(p) > P(p)$

79/5

a) $$\frac{1}{p^2}\cdot e^{-1p} \mathrel{\hat=} (t-1)\cdot H(t-1)$$

b) $$\frac{1}{p+2}\cdot e^{-4p} \mathrel{\hat=} e^{-2\cdot(t-4)}\cdot H(t-4)$$

c) $$\frac{p}{p^2+9}\cdot e^{-\pi p} \mathrel{\hat=} \cos 3(t-\pi)\cdot H(t-\pi) = (-\cos 3t)\cdot H(t-\pi)$$

$\mathcal{L}\{f''(t)\} = p^2 . F(p) - p . \lim f(0+) - \lim f'(0+)$

$\mathcal{L}\{f'(t)\} = p F(p) - \lim f(0+)$

a) $x' + 3x = 0 \qquad x(0+) = r$

$(p.X - r) + 3X = 0$

$X . (p+3) - r = 0$

$$X = \frac{r}{(p+3)} \mathrel{\hat{=}} r . e^{-3t}$$

$$\boxed{x = \left(r e^{-3t}\right) . H(t)}$$

---

b) $x' + 2x = 3 \quad \ldots \quad x(0+) = 0$

$(pX - 0) + 2X = \frac{3}{p}$

$X . (p+2) = \frac{3}{p}$

$$\frac{A}{p} + \frac{B}{p+2}$$

$$A = \frac{3}{(p+2)}\bigg|_{p=0} = \frac{3}{2}$$

$$X = \frac{3}{p . (p+2)} = \frac{3}{2} . \frac{1}{p} - \frac{3}{2} . \frac{1}{(p+2)} \qquad B = \frac{3}{p}\bigg|_{p=-2} = -\frac{3}{2}$$

$$x = \left(\frac{3}{2} - \frac{3}{2} e^{-2t}\right) . H(t) = \boxed{\frac{3}{2} . \left(1 - e^{-2t}\right) . H(t)}$$

---

c) $x' + 2x = \sin t \qquad x(0+) = 0$

$(pX - 0) + 2X = \frac{1}{p^2+1}$

$x . (p+2) = \frac{1}{p^2+1}$

$$\frac{A}{p+2} + \frac{Bx + C}{p^2+1} = \frac{1}{(p^2+1) . (p+2)}$$

$$A = \frac{1}{p^2+1}\bigg|_{p=-2} = \boxed{\frac{1}{5}}$$

$$X = \frac{1}{(p^2+1) . (p+2)} =$$

$A p^2 + A + (Bp + C) . (p+2) = 1$

$p^2 : 0 = A + B \Rightarrow \boxed{B = -\frac{1}{5}}$

$p : 0 = 2B + C \qquad C = -2B$

$p^0 : 1 = A + 2C \qquad \boxed{C = \frac{2}{5}}$

$$= \frac{1}{5} . \frac{1}{p+2} + \frac{-\frac{1}{5}p + \frac{2}{5}}{p^2+1} = \frac{1}{5} . \left(\frac{1}{p+2} + \frac{-p+2}{p^2+1}\right)$$

$$\boxed{\hat{=} \frac{1}{5}\left(e^{-2t} - \cos t + 2 \sin t\right) . H(t)}$$

79/6

d) $x' - 3x = e^{2t}$ $\qquad x(0+) = 1$

$$\frac{1}{(p-3).(p-2)} = \frac{A}{p-3} + \frac{B}{p-2}$$

$$(pX - 1) - 3X = \frac{1}{p-2}$$

$$A = \frac{1}{p-2}\bigg|_{p=3} = 1$$

$$X.(p-3) - 1 = \frac{1}{p-2}$$

$$B = \frac{1}{p-3}\bigg|_{p=2} = -1$$

$$X = \frac{1}{(p-3).(p-2)} + \frac{1}{(p-3)} = \frac{2}{(p-3)} - \frac{1}{(p-2)}$$

$$\underline{x = \left(2e^{3t} - e^{2t}\right).H(t)}$$

---

e) $x'' + x' - 2x = 0$ $\qquad x(0+) = 0 \quad x'(0+) = 3$

$$(p^2X - p0 - 3) + (pX - 0) - 2X = 0$$

$$X.(p^2 + p - 2) - 3 = 0$$

$D = 1 + 8 = 9$

$$X = \frac{3}{p^2 + p - 2} = \frac{3}{(p+2).(p-1)}$$

$p_{1,2} = \frac{-1 \pm 3}{2}$

$$\frac{A}{p+2} + \frac{B}{p-1} = 3$$

$$A = \frac{3}{p-1}\bigg|_{p=-2} = -1$$

$$B = \frac{3}{p+2}\bigg|_{p=1} = 1$$

$$X = \frac{-1}{p+2} + \frac{1}{p-1}$$

$$\underline{x = \left(e^{t} - e^{-2t}\right) H(t)}$$

79/6

f) $x'' - 2x' + 2x = 0 \qquad x(0+) = 1 \qquad x'(0+) = 1$

$$(p^2 X - p1 - 1) - 2(p.X - 1) + 2X = 0$$

$$X \cdot (p^2 - 2p + 2) - p - 1 + 2 = 0$$

$$X = \frac{p-1}{(p^2 - 2p + 1) + 1} = \frac{p-1}{(p-1)^2 + 1}$$

$$x = (e^{t} \cdot \cos t) \cdot H(t)$$

---

g) $x'' + 5x' + 6x = 4e^{-t} \qquad x(0+) = 0 \qquad x'(0+) = 0$

$$(p^2 X - p0 - 0) + 5(pX - 0) + 6X = \frac{4}{p+1}$$

$$X \cdot (p^2 + 5p + 6) = \frac{4}{p+1}$$

$D = 25 - 24 = 1$

$p_{1,2} = \frac{-5 \pm 1}{2} \to -3, \ -2$

$$X = \frac{4}{(p+1).(p^2 + 5p + 6)} = \frac{4}{(p+1).(p+2).(p+3)}$$

$$\frac{4}{(p+1).(p+2).(p+3)} = \frac{A}{p+1} + \frac{B}{p+2} + \frac{C}{(p+3)}$$

$$A = \frac{4}{(p+2).(p+3)}\Big|_{p=-1} = \frac{4}{2} = 2$$

$$B = \frac{4}{(p+1).(p+3)}\Big|_{p=-2} = \frac{4}{-1} = -4$$

$$C = \frac{4}{(p+1).(p+2)}\Big|_{p=-3} = \frac{4}{(-2).(-1)} = 2$$

$$\frac{2}{p+1} - \frac{4}{p+2} + \frac{2}{p+3} \Rightarrow$$

$$x = (2e^{-t} - 4e^{-2t} + 2e^{-3t}) \cdot H(t)$$

79) 6

h) $x'' + x = \sin 2t \qquad x(0+) = 0 \qquad x'(0+) = 0$

$$(p^2 X - p0 - 0) + X = \frac{2}{p^2+4}$$

$$X \cdot (p^2+1) = \frac{2}{p^2+4}$$

$$X = \frac{2}{(p^2+4)\cdot(p^2+1)}$$

$$\frac{2}{(p^2+1)\cdot(p^2+4)} = \frac{Ap+B}{(p^2+1)} + \frac{Cp+D}{p^2+4}$$

$$2 = (Ap+B)\cdot(p^2+4) + (Cp+D)\cdot(p^2+1)$$

$$2 = \cancel{Ap^3} + \cancel{4Ap} + \cancel{Bp^2} + \cancel{4B} + \cancel{Cp^3} + \cancel{Cp} + \cancel{Dp^2} + D$$

p^3: $0 = A + C$

p^2: $0 = B + D \quad = \quad D = -B$

p^1: $0 = 4A + C \quad \underline{D = -\frac{2}{3}}$

p^0: $2 = 4B + D \quad \Rightarrow \quad 3B = 2$

$$\underline{B = \frac{2}{3}}$$

$$\frac{2}{(p^2+4)\cdot(p^2+1)} = \frac{2}{3}\cdot\frac{1}{p^2+1} - \frac{2}{3\cdot 2}\cdot\frac{1\cdot 2}{p^2+2^2}$$

$$\underline{x = \left(\frac{2}{3}\sin t - \frac{1}{3}\sin 2t\right)\cdot \mathbb{1}(t)}$$

79/6

i) $x'' + x = \cos t \qquad x(0+) = -1 \qquad x'(0+) = 1$

$$(p^2 X + p - 1) + X = \frac{p}{p^2+1}$$

$$X \cdot (p^2+1) + p - 1 = \frac{p}{p^2+1}$$

$$X = \frac{p}{(p^2+1)^2} + \frac{1-p}{(p^2+1)}$$

$$x = \left(\frac{1}{2}t \cdot \sin t + \sin t - \cos t\right) \cdot H(t) =$$

$$\boxed{= \left[\left(\frac{1}{2}t + 1\right)\sin t - \cos t\right] \cdot H(t)}$$

---

j) $x'' - 5x' + 4x = 4t^2 e^{2t} \qquad x(0+) = -1 \qquad x'(0+) = 1$

$$(p^2 X + p - 1) - 5 \cdot (pX + 1) + 4X = 4 \cdot \frac{2}{(p-2)^3} =$$

$$X \cdot (p^2 - 5p + 4) + p - 1 - 5 = \frac{8}{(p-2)^3}$$

$D = 25 - 16 = 9$

$p_{1,2} = \frac{+5 \pm 3}{2} \nearrow 4 \searrow 1$

$$X = \frac{8}{(p-2)^3 \cdot (p^2 - 5p + 4)} + \frac{6-p}{(p^2 - 5p + 4)} =$$

$$= \frac{8}{(p-2)^3 \cdot (p-4) \cdot (p-1)} + \frac{6-p}{(p-4) \cdot (p-1)}$$

$\Longrightarrow$

$$\frac{8}{(p-2)^3\cdot(p-4)\cdot(p-1)} = \frac{A}{(p-2)^3} + \frac{B}{(p-2)^2} + \frac{C}{p-2} + \frac{D}{p-4} + \frac{E}{p-1}$$

$$A = \frac{8}{(p-4)\cdot(p-1)}\bigg|_{p=2} = \frac{8}{-2} = \boxed{-4}$$

$$D = \frac{8}{(p-2)^3\cdot(p-1)}\bigg|_{p=4} = \frac{8}{24} = \boxed{\frac{1}{3}}$$

$$E = \frac{8}{(p-2)^3\cdot(p-4)}\bigg|_{p=1} = \frac{8}{(-1)\cdot(-3)} = \boxed{\frac{8}{3}}$$

$$8 = A\cdot(p^2-5p+4) + B\cdot(p-2)\cdot(p^2-5p+4) + C(p-2)^2\cdot(p^2-5p+4) + D\cdot(p-2)^3\cdot(p-1) + E\cdot(p-2)^3\cdot(p-4)$$

$$8 = A(p^2-5p+4) + B\cdot(p^3-7p^2+14p-8) + C(\cancel{p^4}-9p^3+28p^2-36p+16) + D\cdot(\cancel{p^4}-7p^3+18p^2-20p+8) +$$
$$+ E\cdot(\cancel{p^4}-10p^3+36p^2-56p+32)$$

p^4: $0 = C + D + E \Rightarrow C = -D-E = \boxed{-3}$

p^3: $0 = B - 9C - 7D - 10E \Rightarrow B = 9C + 7D + 10E$

$$B = -27 + \frac{7}{3} + \frac{80}{3} = \boxed{2}$$

$$(p-2)\cdot(p^2-5p+4) = p^3 - 5p^2 + 4p - 2p^2 + 10p - 8 = p^3 - 7p^2 + 14p - 8$$
$$(p-2)^2 = p^2 - 4p + 4$$
$$(p-2)^3 = p^3 - 6p^2 + 12p - 8$$
$$(p-2)^2\cdot(p^2-5p+4) = p^4 - 5p^3 + 4p^2 - 4p^3 + 20p^2 - 16p + 4p^2 - 20p + 16 =$$
$$= p^4 - 9p^3 + 28p^2 - 36p + 16$$
$$(p-2)^3(p-1) = p^4 - 6p^3 + 12p^2 - 8p - p^3 + 6p^2 - 12p + 8 =$$
$$= p^4 - 7p^3 + 18p^2 - 20p + 8$$
$$(p-2)^3\cdot(p-4) = p^4 - 6p^3 + 12p^2 - 8p - 4p^3 + 24p^2 - 48p + 32 =$$
$$= p^4 - 10p^3 + 36p^2 - 56p + 32$$

$$\frac{-4}{(p-2)^3} + \frac{2}{(p+2)^2} - \frac{3}{(p-2)} + \frac{1}{3}\cdot\frac{1}{p-4} + \frac{8}{3}\cdot\frac{1}{p-1} \triangleq$$

$\triangleq$

$$\frac{6-p}{(p-4)\cdot(p-1)} = \frac{A}{p-4} + \frac{B}{p-1} \Rightarrow \frac{2}{3}\frac{1}{p-4} - \frac{5}{3}\cdot\frac{1}{p-1}$$

$$A = \left.\frac{6-p}{p-1}\right|_{p=4} = \frac{2}{3}$$

$$B = \left.\frac{6-p}{p-4}\right|_{p=1} = \frac{5}{-3} = -\frac{5}{3}$$

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$$X = \underbrace{\frac{-4}{(p-2)^3} + \frac{2}{(p-2)^2} - \frac{3}{(p-2)} + \left(\frac{1}{3}+\frac{2}{3}\right)\cdot\frac{1}{p-4} + \left(\frac{8-5}{3}\right)\cdot\frac{1}{p-1}}$$

$$x = \left[-2t^2e^{2t} + 2te^{2t} - 3e^{2t} + e^{4t} + e^{t}\right]\cdot H(t) =$$

$$= \underbrace{\left[e^{2t}\cdot\left(-2t^2+2t-3\right) + e^{4t} + e^{t}\right]\cdot H(t)}$$

ŘEŠENÍ 7.9/6j

79/7

a) $x' = x + y \qquad x(0+) = 1$

$y' = -2x + 3y \qquad y(0+) = 1$

$$pX - 1 = X + Y \qquad\qquad X\cdot(p-1) = 1 + Y \quad \Rightarrow \quad Y = X\cdot(p-1) - 1$$
$$pY - 1 = -2X + 3Y \quad \Rightarrow \quad Y\cdot(p-3) = 1 - 2X$$

$$[X\cdot(p-1) - 1]\cdot(p-3) = 1 - 2X$$
$$X\cdot(p^2-4p+3) - (p-3) = 1 - 2X$$
$$X\cdot(p^2-4p+5) = 1 + (p-3)$$
$$X = \frac{p-2}{p^2-4p+5} = \frac{p-2}{(p-2)^2+1}$$

$$Y = \frac{(p-2)\cdot(p-1)}{p^2-4p+5} - \frac{(p^2-4p+5)}{p^2-4p+5} = \frac{\cancel{p^2} - 3p + 2 - \cancel{p^2} + 4p - 5}{p^2-4p+5} =$$
$$= \frac{p-3}{(p^2-4p+5)} = \frac{p-2}{(p-2)^2+1} - \frac{1}{(p-2)^2+1}$$

$$\boxed{\begin{aligned} x &= e^{2t}\cdot\cos t \\ y &= e^{2t}\cdot(\cos t - \sin t) \qquad t \in \langle 0; +\infty) \end{aligned}}$$

73/7

a) $x' = -y \qquad x(0+) = 1$

$y' = 2x + 2y \qquad y(0+) = 1$

$pX - 1 = -Y \quad \rightarrow \quad Y = 1 - pX$

$pY - 1 = 2X + 2Y$

$p\cdot(1 - pX) - 1 = \cancel{2X} + 2 - \cancel{2p}X$

$X\cdot(-p^2 - 2 + 2p) = -p + 1 + 2$

$$X = \frac{p-3}{(p^2 - 2p + 2)} = \frac{p-3}{(p-1)^2 + 1}$$

$$Y = 1 - p\cdot\left(\frac{p-3}{p^2-2p+2}\right) = \frac{\cancel{p^2} - 2p + 2 \cancel{- p^2} + 3p}{p^2 - 2p + 2} = \frac{p+2}{(p-1)^2+1}$$

$$\boxed{\begin{aligned} x &= e^t\cdot(\cos t - 2\sin t) \\ y &= e^t\cdot(\cos t + 3\sin t) \end{aligned} \qquad t \in (0; +\infty)}$$

19/7 d) $\quad x' = x - y + 2e^{t} \qquad x(0+) = 0$

$y' = -x + y + e^{t} \qquad y(0+) = 3$

$$pX - 0 = X - Y + \frac{2}{p-1}$$
$$pY - 3 = -X + Y + \frac{1}{p-1}$$
$$\Rightarrow \quad X\cdot(p-1) = -Y + \frac{2}{p-1} \Rightarrow X = \frac{-Y}{(p-1)} + \frac{2}{(p-1)^2}$$
$$Y\cdot(p-1) = 3 - X + \frac{1}{p-1}$$

$$Y\cdot(p-1) = 3 + \frac{Y}{(p-1)} - \frac{2}{(p-1)^2} + \frac{1}{p-1}$$

$$Y\cdot\left(p-1-\frac{1}{p-1}\right) = \frac{3(p-1)^2 - 2 + (p-1)}{(p-1)^2}$$

$D = 9 - 8 = 1$

$p_{12} = \frac{3 \pm 1}{2} \to 2, \; 1$

$$Y\cdot\left(\frac{(p-1)^2 - 1}{\cancel{p-1}}\right) = \frac{\cancel{3p^2} - \cancel{6p} + 3 - 2 + \cancel{p} - 1}{(p-1)^{\cancel{2}}}$$

$$Y = \frac{3p^2 - 5p}{(p-1)^3 - (p-1)} = \frac{p\cdot(3p-5)}{\cancel{p^3} - \cancel{3p^2} + \cancel{3p} - 1 - p + 1} =$$
$$= \frac{p\cdot(3p-5)}{p^3 - 3p^2 + 2p} = \frac{3p-5}{p^2 - 3p + 2} = \boxed{\frac{3p-5}{(p-2)\cdot(p-1)}}$$

$$A = \frac{3p-5}{(p-1)}\Bigg|_{p=2} = \frac{1}{1} = 1 \quad ; \quad B = \frac{3p-5}{(p-2)}\Bigg|_{p=1} = \frac{-2}{-1} = 2$$

$$\boxed{Y = \frac{1}{p-2} + \frac{2}{p-1}}$$

$$X(p-1) = -\frac{1}{p-2} - \cancel{\frac{2}{p-1}} + \cancel{\frac{2}{p-1}}$$

$$X = -\frac{1}{(p-1)\cdot(p-2)} = \boxed{\frac{1}{(p-1)} - \frac{1}{(p-2)}}$$

$$A = \frac{-1}{(p-2)}\Bigg|_{p=1} = \frac{-1}{-1} = \boxed{1}$$

$$B = \frac{-1}{(p-1)}\Bigg|_{p=2} = \frac{-1}{1} = \boxed{-1}$$

$$\boxed{\begin{aligned} x &= e^{t} - e^{2t} \\ y &= e^{2t} + 2e^{t} \qquad t \in (0, +\infty) \end{aligned}}$$

79/ 7c

$$x' = -x + y + e^t \qquad x(0+) = 0$$
$$y' = x - y + e^t \qquad y(0+) = 0$$

$$pX = -X + Y + \frac{1}{p-1}$$
$$pY = X - Y + \frac{1}{p-1} \Rightarrow \quad X\cdot(p+1) = Y + \frac{1}{p-1}$$
$$Y\cdot(p+1) = X + \frac{1}{p-1} \Rightarrow Y = \frac{X}{(p+1)} + \frac{1}{(p-1)\cdot(p+1)}$$

$$X\cdot(p+1) = \frac{X}{p+1} + \frac{1}{(p+1)\cdot(p-1)} + \frac{1}{p-1}$$
$$X\left(p+1 - \frac{1}{p+1}\right) = \frac{1 + p + 1}{(p+1)\cdot(p-1)}$$
$$X\cdot\left(\frac{(p+1)^2 - 1}{\cancel{(p+1)}}\right) = \frac{p+2}{\cancel{(p+1)}\cdot(p-1)}$$
$$= X = \frac{p+2}{p(p+2)\cdot(p-1)} = \frac{A}{p} + \frac{B}{p+2} + \frac{C}{p-1}$$

$$A = \frac{p+2}{(p+2)\cdot(p-1)}\bigg|_{p=} \frac{2}{-2} = -1 \; ; \; B = \frac{p+2}{p\cdot(p-1)}\bigg|_{p=-2} = 0$$
$$C = \frac{p+2}{p\cdot(p+2)}\bigg|_{p=1} = \frac{3}{3} = 1$$

$$X = -\frac{1}{p} + \frac{1}{p-1}$$

$$Y(p+1) = -\frac{1}{p} + \frac{1}{p-1} + \frac{1}{p-1} \Rightarrow \quad Y = \frac{\overset{\cancel{(1+p)}}{1 - p + 2p}}{p\cdot(p-1)\,\cancel{(p+1)}} = \frac{1}{p\cdot(p-1)}$$

$$A = \frac{1}{p-1}\bigg|_{p=0} = -1 \qquad B = \frac{1}{p}\bigg|_{p=1} = 1$$

$$Y = -\frac{1}{p} + \frac{1}{p-1}$$

$$x = e^t - 1$$
$$y = e^t - 1 \qquad t \in \langle 0; +\infty)$$

79/8

a) $x + \int_0^t x(u)\,du = e^t$

$$X + \frac{X}{p} = \frac{1}{p-1} \Rightarrow X \cdot \left(1 + \frac{1}{p}\right) = \frac{1}{p-1}$$

$$X\left(\frac{p+1}{p}\right) = \frac{1}{p-1}$$

$$X = \frac{p}{p^2-1}$$

$$\underline{x = \cosh t \cdot H(t)}$$

---

b) $x' + 6x + 9\int_0^t x(u)\,du = 0 \qquad x(0+) = 1$

$$pX - 1 + 6X + \frac{9}{p}X = 0$$

$$X \cdot \left(p + 6 + \frac{9}{p}\right) = 1$$

$$X \cdot \left(\frac{p^2+6p+9}{p}\right) = 1$$

$$X = \frac{p}{(p+3)^2} = \frac{A}{(p+3)^2} + \frac{B}{(p+3)} =$$

$$p = A + Bp + 3B$$

p^1: $\underline{1 = B}$

p^0: $0 = A + 3B \Rightarrow A = -3B$

$A = -3$

$$= \underline{\frac{-3}{(p+3)^2} + \frac{1}{(p+3)}}$$

$$x = -3te^{-3t} + 1e^{-3t} = \underline{e^{-3t}(1-3t) \cdot H(t)}$$

79) ℓc $\quad x' + 2x + 5\int_0^t x(u)\,du = 0 \qquad x(0+) = -1$

$$pX + 1 + 2X + \frac{5}{p}X = 0$$

$$X\cdot\left(p + 2 + \frac{5}{p}\right) = -1 \qquad D = 4 - 20 < 0$$

$$X\left(p^2 + 2p + 5\right) = -p$$

$$X = \frac{-p}{p^2+2p+5} = \frac{-(+p+1)}{(p+1)^2+2^2} + \frac{1}{2}\frac{1\cdot 2}{(p+1)^2+2^2}$$

$$x = -e^{-t}\cdot\cos 2t + \frac{e^{-t}}{2}\cdot\sin 2t = \boxed{e^{-t}\cdot\left(\frac{1}{2}\sin 2t - \cos 2t\right)\cdot H(t)}$$

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2) $\quad x' - 4x + 5\int_0^t x(u)\,du = 2e^t \qquad x(0+) = 1$

$$pX - 1 - 4X + \frac{5}{p}X = \frac{2}{p-1} \qquad D = 16 - 20 < 0$$

$$X\cdot\left(\frac{p^2-4p+5}{p}\right) = \frac{2}{p-1} + \frac{p-1}{p-1}$$

$$X = \frac{p\cdot(p+1)}{(p^2-4p+5)\cdot(p-1)} = \frac{A}{(p-1)} + \frac{Bp+C}{p^2-4p+5}$$

$$p^2 + p = Ap^2 - 4Ap + 5A + Bp^2 - Bp + Cp - C$$

p^2: $\quad 1 = A + B \quad \to \quad A = 1 - B \quad \to \quad \boxed{A = 1}$

p^1: $\quad +1 = -4A - B + C$

p^0: $\quad 0 = 5A - C \quad \to \quad +1 = -4 + 4B - B + C$

$\quad C = 5 \qquad\qquad 0 = 5 - 5B - C$

$$+1 = 1 - 2B \to \boxed{B = 0}$$

$$X = \frac{1}{p-1} + \frac{5}{(p-2)^2+1^2} = \frac{1}{p-1} + 5\cdot\frac{1}{(p-2)^2+1}$$

$$\boxed{x = \left[e^{t} + 5\cdot e^{2t}\cdot\sin t\right]\cdot H(t)}$$

10/2e

$$x' + \int_0^t \cosh(t-u)\cdot x(u)\,du = e^{-t} \qquad x(0+) = 0$$

$$pX + X\cdot\left(\frac{p}{p^2-1}\right) = \frac{1}{p+1}$$

$$X\cdot\left(p + \frac{p}{p^2-1}\right) = \frac{1}{p+1}$$

$$X\cdot\left(\frac{p^3 \cancel{-p+p}}{\cancel{(p+1)}\cdot(p-1)}\right) = \frac{1}{\cancel{p+1}}$$

$$X = \frac{p-1}{p^3} = \frac{1}{p^2} - \frac{1}{p^3}$$

$$\boxed{x(t) = \left(t - \frac{1}{2}t^2\right)\cdot H(t)}$$

8f

$$x' + 2x + 2\cdot\int_0^t e^{(t-u)}\, x(u)\,du = e^t \qquad x(0+) = 1$$

$$(pX - 1) + 2X + 2\cdot\left(X\cdot\frac{1}{p-1}\right) = \frac{1}{p-1}$$

$$pX - 1 + 2x + \frac{2x}{p-1} = \frac{1}{p-1}$$

$$x\cdot\left(p + 2 + \frac{2}{p-1}\right) = \frac{1}{p-1} + 1$$

$$X\cdot\left(\frac{((p+2)(p-1)+2)}{\cancel{p-1}}\right) = \frac{1+p-1}{\cancel{p-1}}$$

$$x\cdot(p^2 + 2p - p \cancel{-2+2}) = p$$

$$x\cancel{p}(p+1) = \cancel{p}$$

$$x = \frac{1}{(p+1)}$$

$$\boxed{x(t) = e^{-t}\cdot H(t)}$$

10/9

a)

$$x' - x = \begin{cases} 2-t & t \in (0;2) \\ 0 & t \geq 2 \end{cases} \qquad x(0+) = -1$$

$$(2-t)\cdot[H(t) - H(t-2)] = (2-t)\cdot H(t) + (t-2)\cdot H(t-2) \triangleq \underbrace{\frac{2}{p} - \frac{1}{p^2} + \frac{1}{p^2}\cdot e^{-2p}}$$

$$pX + 1 - X = \frac{2p-1}{p^2} + \frac{e^{-2p}}{p^2}$$

$$X = \frac{2p-1}{p^2(p-1)} + \frac{e^{-2p}}{p^2(p-1)} - \frac{p^2}{p^2(p-1)} =$$

$$= \frac{-(p^2-2p+1)}{p^2(p-1)} + \frac{e^{-2p}}{p^2(p-1)} = -\frac{(p-1)}{p^2} + \frac{1}{p^2(p-1)}\cdot e^{-2p}$$

$$= \underbrace{-\frac{1}{p} + \frac{1}{p^2} + \left(-\frac{1}{p^2} - \frac{1}{p} + \frac{1}{p-1}\right)\cdot e^{-2p}}$$

$$\frac{1}{p^2(p-1)} = \frac{A}{p^2} + \frac{B}{p} + \frac{C}{p-1}; \quad A = -1$$

$$B = -C = -1$$

$$1 = A(p-1) + Bp(p-1) + Cp^2 \qquad C = 1$$

$$\mathcal{L}^{-1}\{X\} = (t-1)[H(t) - H(t-2)] + \left(-(t-2) - 1 + e^{(t-2)}\right)\cdot H(t-2)$$

$$= (t-1)\cdot H(t) + (1-t)H(t-2) + \left(-(t-2) - 1 + e^{t-2}\right)\cdot H(t-2)$$

$$1 - t = -t + 2 - 1 + e^{t-2}$$

$$\cancel{1-t} = \cancel{1-t} + \underbrace{e^{t-2}}$$

$$\boxed{x(t) = \begin{cases} t-1 & ; t \in [0;2) \\ e^{t-2} & ; t \in [2;\infty) \end{cases}}$$

10) 4)

$$x' + x = \begin{cases} 5\cos 2t & t \in \langle 0; \frac{\pi}{2}) \\ 0 & t \geq \frac{\pi}{2} \end{cases} \qquad x(0+) = 1$$

$$5\cos 2t \cdot \left[H(t) - H\left(t - \tfrac{\pi}{2}\right)\right] = 5\cos 2t \cdot H(t) - 5\left[\cos\left(2\cdot\left(t - \tfrac{\pi}{2}\right) + \pi\right) \cdot H(t-2)\right] =$$

$$= 5\cdot\cos 2t \cdot H(t) - \left[5\cos(2t - \pi)\cdot \underbrace{\cos\pi}_{(-1)} - \cancel{\sin(2t-\pi)\cdot \underbrace{\sin\pi}_{0}}\right]\cdot H\left(t - \tfrac{\pi}{2}\right) =$$

$$= 5\cdot\cos 2t\cdot H(t) + \cos 2\cdot\left(t - \tfrac{\pi}{2}\right)\cdot H\left(t - \tfrac{\pi}{2}\right) =$$

$$\hat{=}\ 5\cdot\frac{p}{p^2+4} + \frac{5\cdot p}{p^2+4}\cdot e^{-\frac{\pi}{2}p} = \boxed{\frac{5p}{(p^2+4)}\cdot\left(1 + e^{-\frac{\pi}{2}p}\right)}$$

$$pX - 1 + X = \frac{5p}{(p^2+4)}\cdot\left(1 + e^{-\frac{\pi}{2}p}\right)$$

$$p^2 + 5p + 4 = 0 \qquad D = 25 - 16$$

$$p_{1,2} = \frac{-5 \pm 4}{2} \begin{cases} -\frac{1}{2} \\ -\frac{9}{2} \end{cases}$$

$$(p+1)\cdot\left(p + \tfrac{9}{2}\right)$$

$$X = \frac{5p}{(p^2+4)\cdot(p+1)}\cdot\left(1 + e^{-\frac{\pi}{2}p}\right) + \frac{1}{p+1} =$$

$$\frac{5p}{(p^2+4)(p+1)} = \frac{A}{p+1} + \frac{Bp + C}{p^2+4} \quad ; \quad A = \frac{5p}{p^2+4}\Big|_{p=-1} = \frac{-5}{5} = \underline{-1}$$

$$5p = A(p^2+4) + Bp^2 + Cp + Bp + C$$

$$p^2:\ 0 = A + B \Rightarrow B = 1$$

$$p^1:\ 5 = C + B \Rightarrow C = 5 - B = +4$$

$$X = \frac{1}{p+1} + \left(-\frac{1}{p+1} + \frac{p+4}{p^2+4}\right) + e^{-\frac{\pi}{2}p}\cdot\left(\frac{p+4}{p^2+4} - \frac{1}{p+1}\right) =$$

$$= \frac{p}{p^2+(2)^2} + \frac{4}{(p^2)+(2)^2} + e^{-\frac{\pi}{2}p}\left(\frac{p}{p^2+2^2} + 2\cdot\frac{2}{p^2+2^2} - \frac{1}{p+1}\right)$$

$$\mathcal{L}^{-1}\{X\} = (\cos 2t + 2\sin 2t)\cdot H(t) + \left(\cos 2(t-\pi) + 2\sin 2(t-\pi) - e^{-(t-\frac{\pi}{2})}\right) H\left(t - \tfrac{\pi}{2}\right)$$

$$= (\cos 2t + 2\sin 2t)\left[H(t) - H\left(t - \tfrac{\pi}{2}\right)\right] + \ldots$$

$$\Rightarrow (\cos 2t + 2\sin 2t)\cdot H(t) - \left(e^{\frac{\pi}{2} - t}\right)\cdot H\left(t - \tfrac{\pi}{2}\right)$$

$$\boxed{X(t) = \begin{cases} \cos 2t + 2\sin 2t \ ; & t \in \langle 0; \frac{\pi}{2}) \\ -e^{+\frac{\pi}{2} - t} & t \geq \frac{\pi}{2} \end{cases}}$$

10/9 c) $x'' + x = \begin{cases} 1 & t \in \langle 0; \pi) \\ 0 & t \geq \pi \end{cases}$ $\quad x(0+) = 0$, $x'(0+) = 0$

$$1 \cdot [H(t) - H(t-\pi)] = 1 \cdot H(t) - 1 H(t-\pi) \hat{=} \frac{1}{p} - \frac{1}{p} \cdot e^{-\pi p}$$

$$p^2 X + X = \frac{1}{p} \cdot (1 - e^{-\pi p})$$

$$X = \frac{1}{(p^2+1) \cdot p} \cdot (1 - e^{-\pi p})$$

$$\frac{1}{p \cdot (p^2+1)} = \frac{A}{p} + \frac{Bp + C}{p^2+1}$$

$$1 = Ap^2 + A + Bp^2 + Cp$$

$0 = A + B \Rightarrow B = -1$

$A = 1 \qquad C = 0$

$$X = \left(\frac{1}{p} - \frac{p}{p^2+1}\right) \cdot (1 - e^{-\pi p})$$

$$f(t) = (1 - \cos t) \cdot H(t) + (-1 + \cos(t-\pi)) \cdot H(t-\pi)$$

$$\boxed{f(t) = \begin{cases} (1 - \cos t) & t \in \langle 0; \pi) \\ (1 - \cos t - 1 + \cos(t-\pi)) = -2\cos t & ; t \in \langle \pi; +\infty) \end{cases}}$$

d) $x'' + 4x = \begin{cases} 4 & t \in (0; \frac{\pi}{4}) \\ 8 & t \in \langle \frac{\pi}{4}; +\infty) \end{cases}$ $\quad x(0+) = 4$, $\quad x'(0+) = 0$

$$p^2 X - 4p + 4X = 4 \cdot [H(t) - H(t - \tfrac{\pi}{4})] + 8 \cdot [H(t - \tfrac{\pi}{4})]$$

$$p^2 X - 4p + 4X = 4H(t) + 4H(t - \tfrac{\pi}{4})$$

$$p^2 X - 4p + 4X = \frac{4}{p} + \frac{4}{p} e^{-\frac{\pi}{4} p}$$

$$X \cdot (p^2 + 4) \cancel{- 4p} = \frac{4}{p} \cdot \left(1 + e^{-\frac{\pi}{4} p}\right) + 4p$$

$$X = \frac{4}{p \cdot (p^2 + 4)} \cdot \left(1 + e^{-\frac{\pi}{4} p}\right) + \frac{4p}{p^2 + 4}$$

$$\frac{1}{p \cdot (p^2 + 4)} = \frac{A}{p} + \frac{Bp + C}{p^2 + 4} \quad ; \quad A = \frac{1}{p^2 + 4}\Big|_{p=0} = \frac{1}{4}$$

$$1 = Ap^2 + 4A + Bp^2 + Cp$$

p^2: $0 = A + B \Rightarrow B = -\frac{1}{4}$

$C = 0$

$$X = 4 \cdot \left(\frac{1}{4p} - \frac{\frac{1}{4} p}{p^2 + 4} \right) \cdot \left(1 + e^{-\frac{\pi}{4} p}\right) + 4 \cdot \frac{p}{p^2 + 2^2}$$

$$= \left(\frac{1}{p} - \frac{p}{p^2 + 2^2} \right) \cdot \left(1 + e^{-\frac{\pi}{4} p}\right) + 4 \cdot \frac{p}{p^2 + 2^2}$$

$$x(t) = (1 - \cos 2t) \cdot H(t) + 4 \cdot \cos 2t \cdot H(t) + \left[+ (1 - \cos 2t) \cdot H(t - \tfrac{\pi}{4}) \right]$$

$$= \boxed{(3 \cos 2t + 1) \cdot H(t) + (-\cos 2t + 1) \cdot H(t - \tfrac{\pi}{4})}$$

$$3 \cos 2t + 1 + \cos 2(t - \tfrac{\pi}{4}) + 1 = 3 \cos 2t - \sin 2t$$

$$\boxed{x(t) = \begin{cases} 3 \cos 2t + 1 & ; t \in \langle 0; \frac{\pi}{4}) \\ 3 \cos 2t - \sin 2t + 2 & ; t \geq \frac{\pi}{4} \end{cases}}$$?

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$$x'' + 9x = \begin{cases} \sin t & ; t \in \langle 0; \pi) \\ 0 & t \geq \pi \end{cases} \qquad \begin{aligned} x(0+) &= 0 \\ x'(0+) &= 0 \end{aligned}$$

$$\sin t \cdot [H(t) - H(t-\pi)] = \sin t \cdot H(t) - \sin t \cdot H(t-\pi) =$$

$$\doteq \frac{p}{p^2+1}\left(1 - e^{-\pi p}\right)$$

$$p^2 X + 9X = \frac{p}{p^2+1}\left(1 - e^{-\pi p}\right)$$

$$X = \frac{p}{(p^2+1)\cdot(p^2+9)} \cdot \left(1 - e^{-\pi p}\right)$$

$$\frac{8}{(p^2+1)\cdot(p^2+9)} = \frac{Ap+B}{p^2+1} + \frac{Cp+D}{p^2+9} \Rightarrow \frac{1}{p^2+1} - \frac{1}{p^2+9}$$

$$8 = (Ap+B)(p^2+9) + (Cp+D)\cdot(p^2+1)$$

$$8 = \cancel{Ap^3} + \cancel{Bp^2} + \cancel{9A}p + \cancel{9B} + \cancel{Cp^3} + \cancel{Cp} + \cancel{Dp^2} + \cancel{D}$$

p^3: $0 = A + C$

p^2: $0 = B + D \Rightarrow B = -D \Rightarrow D = -1$

p^1: $0 = 9A + C$

p^0: $8 = 9B + D \Rightarrow \boxed{8 = 8B = 1}$

$$X = \left(\frac{1}{p^2+1} - \frac{1}{p^2+9}\right)\cdot\left(1 - e^{-\pi p}\right) = \left(\frac{1}{p^2+1} - \frac{1}{3}\frac{3}{p^2+3^2}\right)\cdot\left(1 - e^{-\pi p}\right)$$

$$= \left(\sin t - \frac{1}{3}\sin 3t\right)\cdot H(t) + \left[\frac{1}{3}\sin 3(t-\pi) - \sin(t-\pi)\right] H(t-\pi)$$

$$\sin t - \frac{1}{3}\sin 3t + \frac{1}{3}\sin 3t - \sin t = \boxed{0}$$

$$\boxed{x(t) = \sin t - \frac{1}{3}\sin 3t \; ; t \in \langle 0; \pi)}$$

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$$x'' - x = \begin{cases} 1-t & ; t \in \langle 0;1) \\ 0 & t \geq 1 \end{cases} \qquad x(0+) = 0, \quad x'(0+) = 0$$

$$(1-t)\cdot\left[H(t) - H(t-1)\right] = (1-t)\,H(t) + (t-1)\cdot H(t-1) \risingdotseq$$

$$\risingdotseq \frac{1}{p} - \frac{1}{p^2} + \frac{1}{p^2}\cdot e^{-p} - \frac{1}{p}\cdot e^{-p} = \left(\frac{1}{p} - \frac{1}{p^2}\right)\cdot\left(1 - e^{-p}\right)$$

$$p^2 X - X = \left(\frac{1}{p} - \frac{1}{p^2}\right)\cdot\left(1 - e^{-p}\right)$$

$$X\cdot\left(p^2 - 1\right) = \left(\frac{p-1}{p^2}\right)\cdot\left(1 - e^{-p}\right)$$

$$X = \frac{p-1}{p^2\cdot(p^2-1)}\cdot\left(1 - e^{-p}\right)$$

$$\frac{p-1}{p^2\cdot(p^2-1)} = \frac{A}{p^2} + \frac{B}{p} + \frac{Cp+D}{p^2-1} \; ; \qquad A = \left.\frac{p-1}{p^2-1}\right|_{p=0} = \frac{-1}{-1} = \boxed{1}$$

$$p - 1 = \cancel{Ap^2} - A + \cancel{Bp^3} - \cancel{Bp} + \cancel{Cp^3} + \cancel{Dp^2}$$

$$p^3 : \; 0 = B + C \Rightarrow \boxed{C = 1}$$

$$p^2 : \; 0 = A + D \Rightarrow \boxed{D = -1}$$

$$p^1 : \; +1 = -B \Rightarrow \boxed{B = -1}$$

$$\frac{1}{p^2} - \frac{1}{p} + \frac{p-1}{p^2-1} = \frac{1}{p^2} - \frac{1}{p} + \frac{\cancel{(p-1)}}{\cancel{(p-1)}\cdot(p+1)} \Rightarrow \boxed{\frac{1}{p^2} - \frac{1}{p} + \frac{1}{p+1}}$$

$$X = \left(\frac{1}{p^2} - \frac{1}{p} + \frac{1}{p+1}\right)\cdot\left(1 - e^{-p}\right)$$

$$\left(t - 1 + e^{-t}\right)H(t) - \left(\cancel{t-1} + \cancel{1} = 1, + e^{-(t-1)}\right) =$$

$$= e^{-t} - e^{-(t-1)}$$

$$x(t) = \begin{cases} t - 1 + e^{-t} & ; t \in \langle 0;1) \\ e^{-t} - e^{1-t} & ; t \geq 1 \end{cases}$$